Fake Supergravity

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Supersymmetry

- Space-time symmetry algebra (Poincaré) extended with fermionic (anticommuting) generators $\mathcal{Q}_{i\alpha}$, $i = 1, \ldots, \mathcal{N}$
- The $\mathcal{Q}$ generators transform as spinors under the Poincaré group, and

$$\{\mathcal{Q}, \mathcal{Q}\} \propto P$$

- Action on *B*osonic and *F*ermionic fields:

$$\delta_{\mathcal{Q}}(\epsilon) B \sim \epsilon F, \quad \delta_{\mathcal{Q}}(\epsilon) F \sim \epsilon B$$

- Every bosonic field must have a fermionic *superpartner* and viceversa

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- Every bosonic field must have a fermionic *superpartner* and viceversa
- The supersymmetry algebra may be further extended with internal symmetry generators T_A (R-symmetry), with

$$[T, \mathcal{Q}] \propto \mathcal{Q}$$

Supergravity

- We could require a theory to be invariant under local supersymmetry transformations: $\epsilon \to \epsilon(x)$
- Since $\{\mathcal{Q}, \mathcal{Q}\} \propto P$ the theory must also be invariant under local translations (diffeomorphisms)
- General relativistic theory: includes gravity $\Longrightarrow$ supergravity

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Example: $\mathcal{N} = 1$ minimal supergravity in 4 dimensions

$$e^{-1}\mathcal{L} = R - \bar{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu \psi_\rho$$

$$\delta e^a_\mu = \frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu, \qquad \delta\psi_\mu = D_\mu\epsilon \equiv \partial_\mu\epsilon + \frac{1}{4}\omega_{\mu ab}\gamma^{ab}\epsilon$$

Solving einstein equations

Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu} - \Lambda g_{\mu\nu}$$

- 10 second order partial differential equations
- Highly coupled
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Finding analytical solutions is not an easy task

Usually solved by first imposing some symmetry on the solution

Supersymmetric solutions

If we consider a supergravity theory, we can look for supersymmetric solutions:

- purely bosonic → they are solutions both of the full theory and of the truncated bosonic sector
- invariant under some (global) supersymmetry transformation

The supersymmetry variation of such field configurations must vanish

- The variation of the bosonic fields automatically vanishes: $\delta_{\mathcal{Q}}(\epsilon)B \sim \epsilon F = 0$
- For the fermionic fields we must impose $\delta_{\mathcal{Q}}(\epsilon)F = 0$

Killing spinor equations

In any supergravity there is at least one gravitino field

- Gravitino condition: $\delta_{\mathcal{Q}}(\epsilon)\psi^{i}_{\mu} = \hat{D}_{\mu}\epsilon^{i} = 0$
- Other fermionic fields: $\delta_{\mathcal{Q}}(\epsilon)\zeta^{A} = 0, \dots$

These conditions depend on the bosonic fields and are called Killing spinor equations; the solutions $\epsilon^{i}(x)$ are Killing spinors.

- First order differential equations $\rightarrow$ easier to solve
- The integrability conditions impose strong constraints on the field configuration $\rightarrow$ usually they imply the equations of motion

Fake Supergravity

Can this method be generalized to theories that aren't truncations of supersymmetric ones?

e.g.: typically SUGRAs admit only negative or vanishing cosmological constant

Old idea: pre-WWII "Wave Geometry" program in Hiroshima

- Inspired by Dirac's equation as "square root" of Klein-Gordon equation
- Attempt to classify all spacetime metrics admitting a nonzero spinor k satisfying

  $$(D_\mu + M_\mu)k = 0$$

  for all possible choices of M_μ.

Fake $\mathcal{N} = 2$ gauged pure supergravity

$\mathcal{N} = 2$ pure supergravity in 4d with gauged $U(1)$ R-symmetry subgroup

Bosonic lagrangian: $e^{-1}\mathcal{L}_{Bos.} = \frac{1}{2}R - \frac{1}{2}F_{\mu\nu}F^{\mu\nu} + 3g^2$

KSEs: $D_\mu \epsilon_i = -\frac{g}{2}\gamma_\mu \varepsilon_{ij}\epsilon^j - igA_\mu \epsilon_i + iF^+_{\mu\nu}\gamma^b \varepsilon_{ij}\epsilon^j$

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$\mathcal{N} = 2$ *fake* pure supergravity in 4d with gauged $\mathbb{R}$-symmetry

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The Kastor-Traschen solution

(D. Kastor and J.H. Traschen 1993)

$$ds^2 = -U^{-2}dt^2 + U^2 d\mathbf{r}^2, \qquad A = U^{-1}dt$$

$$U = \frac{t}{t_0} + \sum_{i=1}^{n} \frac{Q_i}{|\mathbf{r} - \mathbf{r}_i|}$$

- Multi-black hole solution
- Cosmological
- Time-dependent

Fake $\mathcal{N}=2$ gauged sugra coupled to vector multiplets

Bosonic lagrangian:

$$e^{-1}\mathcal{L}_{Bos.} = \frac{1}{2}R - g_{i\bar{\jmath}}\mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{\jmath}} - V$$
$$+ \mathfrak{Im}\,(\mathcal{N})_{\Lambda\Sigma}\, F^\Lambda_{\mu\nu} F^{\Sigma\mu\nu} - \mathfrak{Re}\,(\mathcal{N})_{\Lambda\Sigma}\, F^\Lambda_{\mu\nu} \star F^{\Sigma\mu\nu}$$

fake KSEs:

$$D_\mu\epsilon_I + \frac{i}{2}\mathcal{Q}_\mu\epsilon_I + \frac{ig}{2}A^\Lambda_\mu(P_\Lambda + iC_\Lambda)\epsilon_I = -2i\mathcal{L}_\Lambda F^{\Lambda+}_{\mu\nu}\gamma^\nu\varepsilon_{IJ}\epsilon^J - \frac{ig}{4}C_\Lambda\mathcal{L}^\Lambda\gamma_\mu\varepsilon_{IJ}\epsilon^J$$

$$i\not{\partial} z^i\epsilon^I = \bar{f}^i_\Lambda \not{F}^{\Lambda+}\varepsilon^{IJ}\epsilon_J + \frac{ig}{2}\bar{f}^{i\Lambda}(P_\Lambda + iC_\Lambda)\varepsilon^{IJ}\epsilon_J$$

- $\mathbb{R}$-symmetry gauged with connection $C_\Lambda A^\Lambda$
- The lagrangian can be completely determined by specifying a *prepotential* and the constants C_Λ
- Fake supersymmetric solutions: general method by P. Meessen and A. Palomo-Lozano (2009)

Einstein-Maxwell-Scalar fake supersymmetric solutions

$$e^{-1}\mathcal{L} = \frac{1}{2}\Big\{ R - \frac{n_0 n_1}{8}\partial_\mu\phi\partial^\mu\phi - \frac{n_0}{4}e^{\frac{n_1}{2}\phi}F^0_{\mu\nu}F^{0\mu\nu} - \frac{n_1}{4}e^{-\frac{n_0}{2}\phi}F^1_{\mu\nu}F^{1\mu\nu}$$
$$- \frac{1}{2}\left[\frac{n_0(n_0-1)}{t_0^2}e^{-\frac{n_1}{2}\phi} + 2\frac{n_0 n_1}{t_0 t_1}e^{\frac{n_0-n_1}{4}\phi} + \frac{n_1(n_1-1)}{t_1^2}e^{\frac{n_0}{2}\phi}\right]\Big\}$$

$$ds^2 = -U^{-2}dt^2 + U^2 d\mathbf{r}^2, \qquad U = H_0^{\frac{n_0}{4}}H_1^{\frac{n_1}{4}} \equiv \left[\frac{t}{t_0} + \mathcal{H}_0\right]^{\frac{n_0}{4}}\left[\frac{t}{t_1} + \mathcal{H}_1\right]^{\frac{n_1}{4}}$$

$$\phi = \log\frac{H_0}{H_1}, \qquad A^0 = H_0^{-1}dt, \qquad A^1 = H_1^{-1}dt$$

- One abelian vector multiplet
- No additional gaugings
- Prepotential: $F = -\frac{i}{4}(X^0)^{\frac{n_0}{2}}(X^1)^{\frac{n_1}{2}}$, $n_0 + n_1 = 4$
- Generalization of the solutions by G.W. Gibbons and K.I. Maeda (2010) ($t_0 \to \infty$ or $t_1 \to \infty$)

Conclusions

Summary:

- (Very) brief introduction to supersymmetry and supergravity
- Supersymmetric solutions and Killing spinor equations
- Generalization: fake supergravity
- Example: Kastor-Traschen solution from $\mathcal{N} = 2$ gauged fake sugra
- Example: our solution

Outlook:

- Study physical properties of our solution
- Generalize to other models
- Look for multi-black hole solutions in non-flat FLRW cosmologies