



CENTER FOR
COMPLEXITY
& BIOSYSTEMS

University of Milan



University of Milan
Department of Physics

Overshoot of cancer stem cell population

A theoretical model via rate eq.
and stochastic simulations

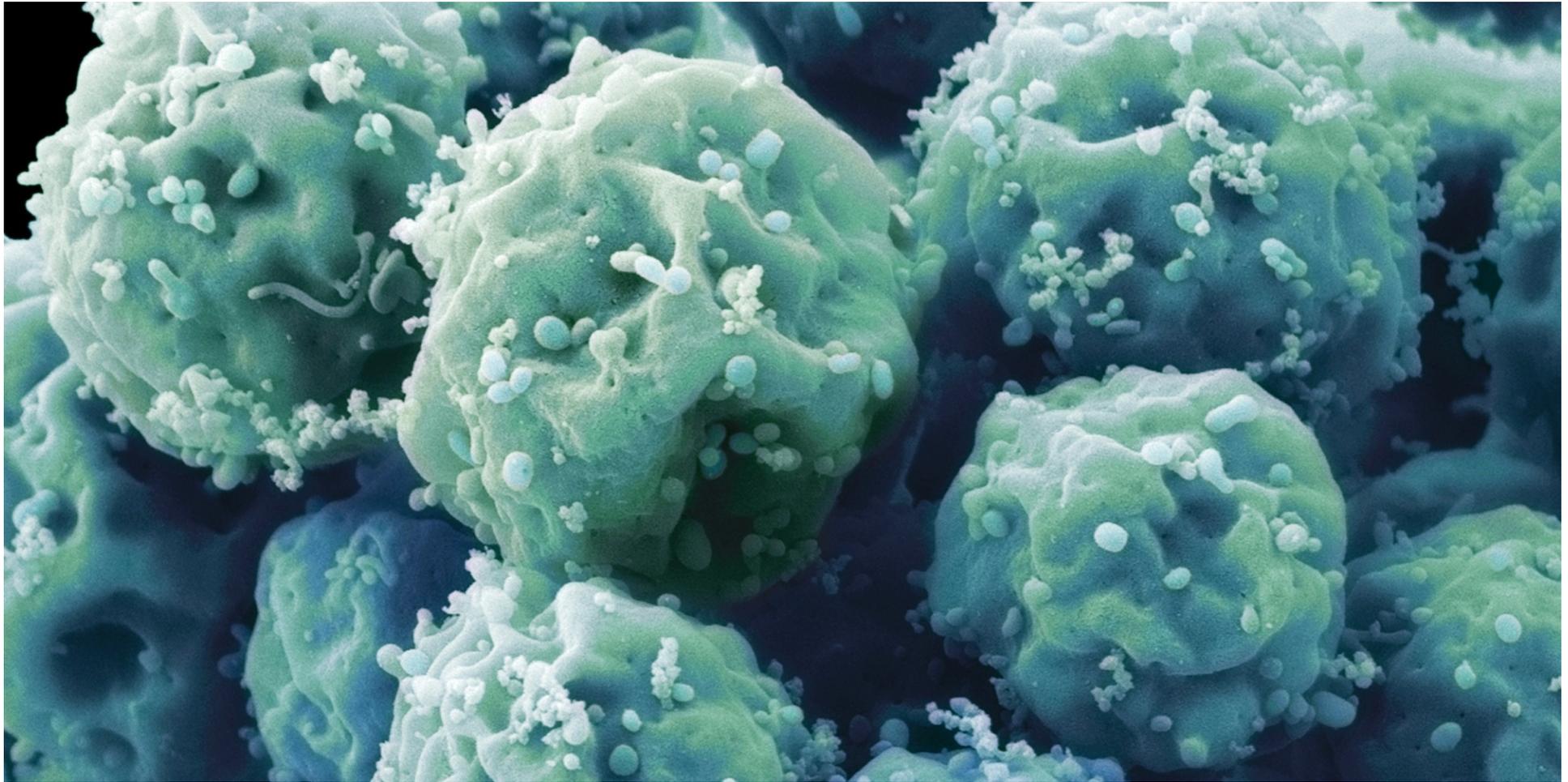
21th October
2016

-Milan

Filippo Cola

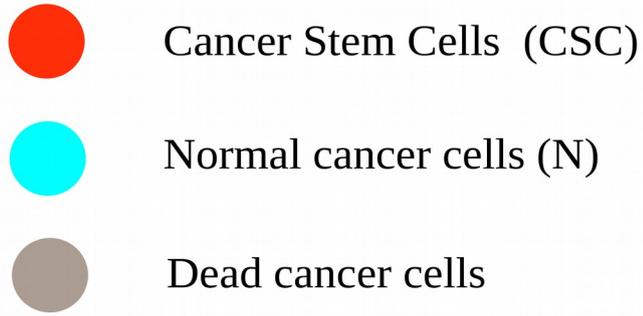
WWW.COMPLEXITY.UNIMI.IT

Unusually for physicists, the main protagonist of our talk shall be biological cells... in particular
CANCER STEM CELLS (CSC)

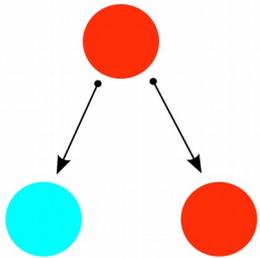


The Bayer Scientific Magazine

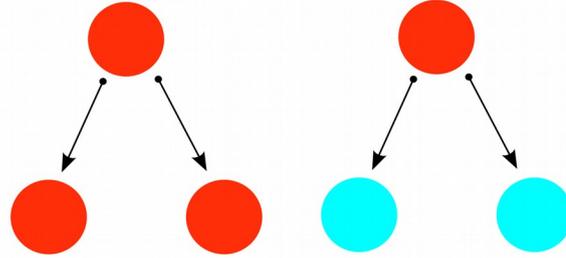
Aberrant CSC Hierarchy Theory for Tumor Growth



Asymmetric division



Symmetric division



Death of N cells



Phenotypic switching

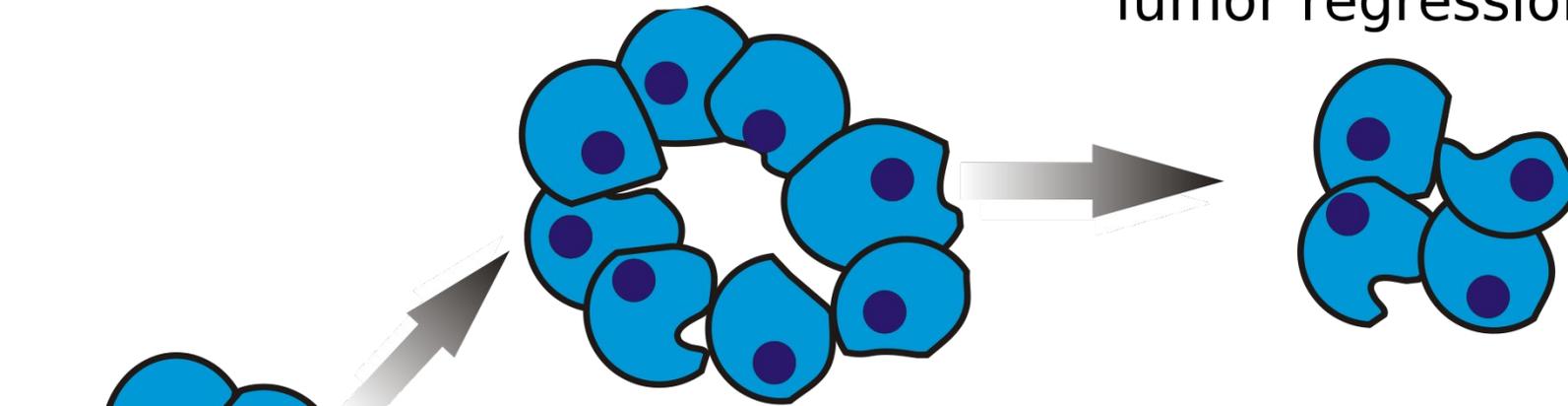


- only small fraction of malignant cells drives tumor growth (stem cell like);
- CSC prop.: self-renewal, differentiation into multiple cell types, longer life span;
- origin not clear;
- first identified in leukemia, then also in brain, breast, colon, pancreas cancers and melanoma.

The hierarchy model suggests a specific therapy to eradicate CSC population

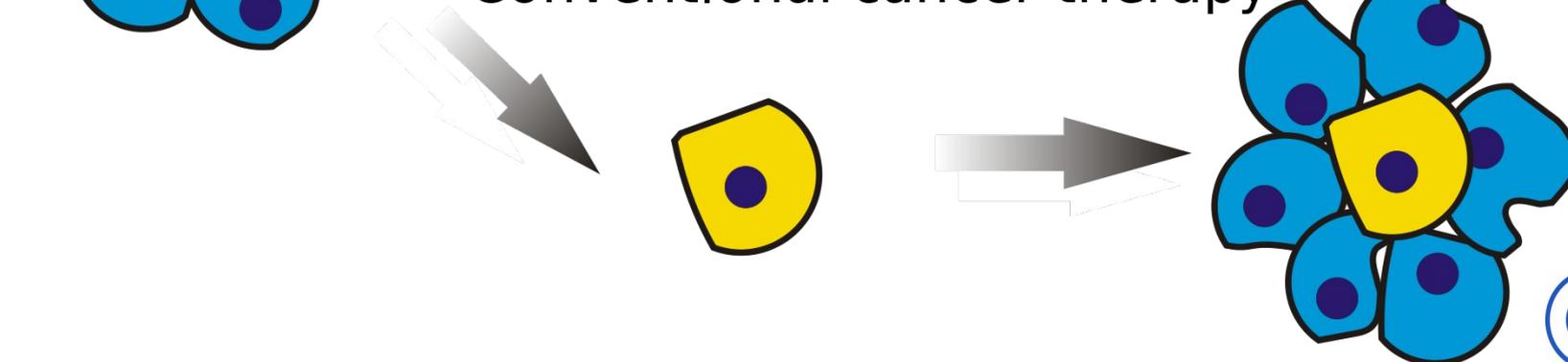
Cancer stem cells specific therapy

Tumor regression



Conventional cancer therapy

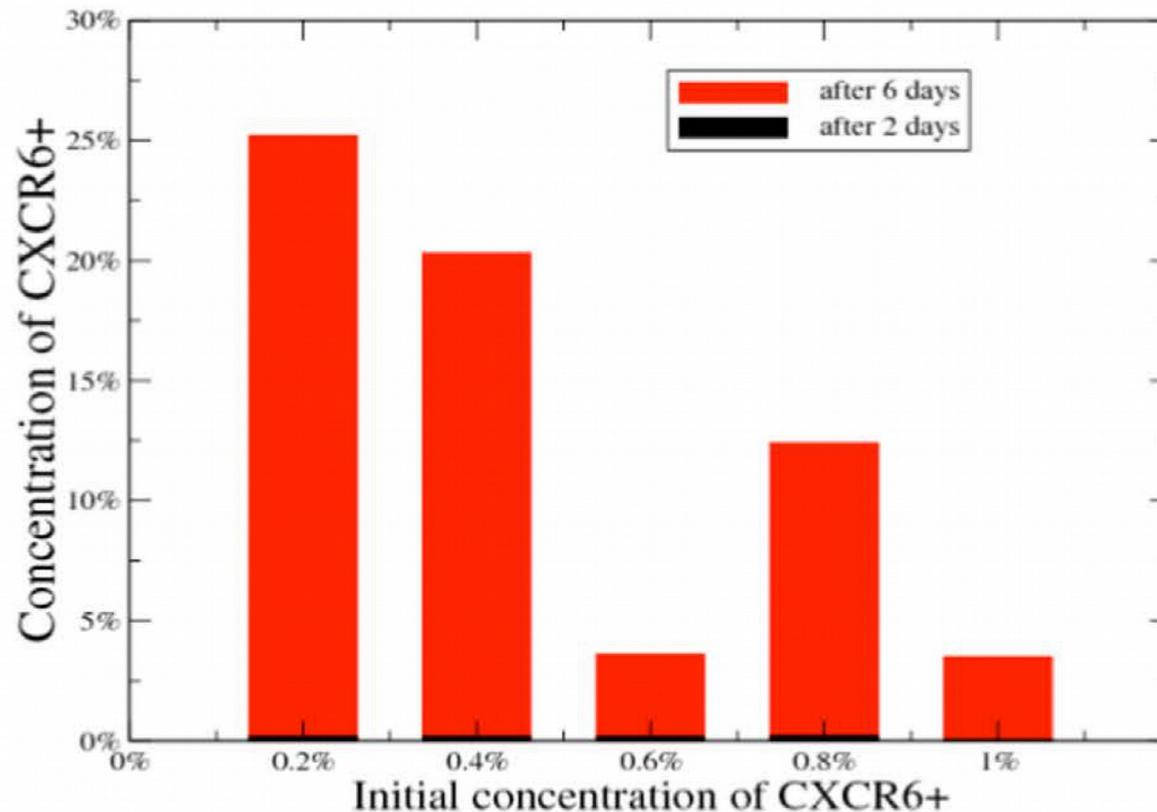
Tumor relapse



https://en.wikipedia.org/wiki/Cancer_stem_cell

However, experiments performed by **La Porta's Group** in the Department of Biosciences suggest this may not always be the case

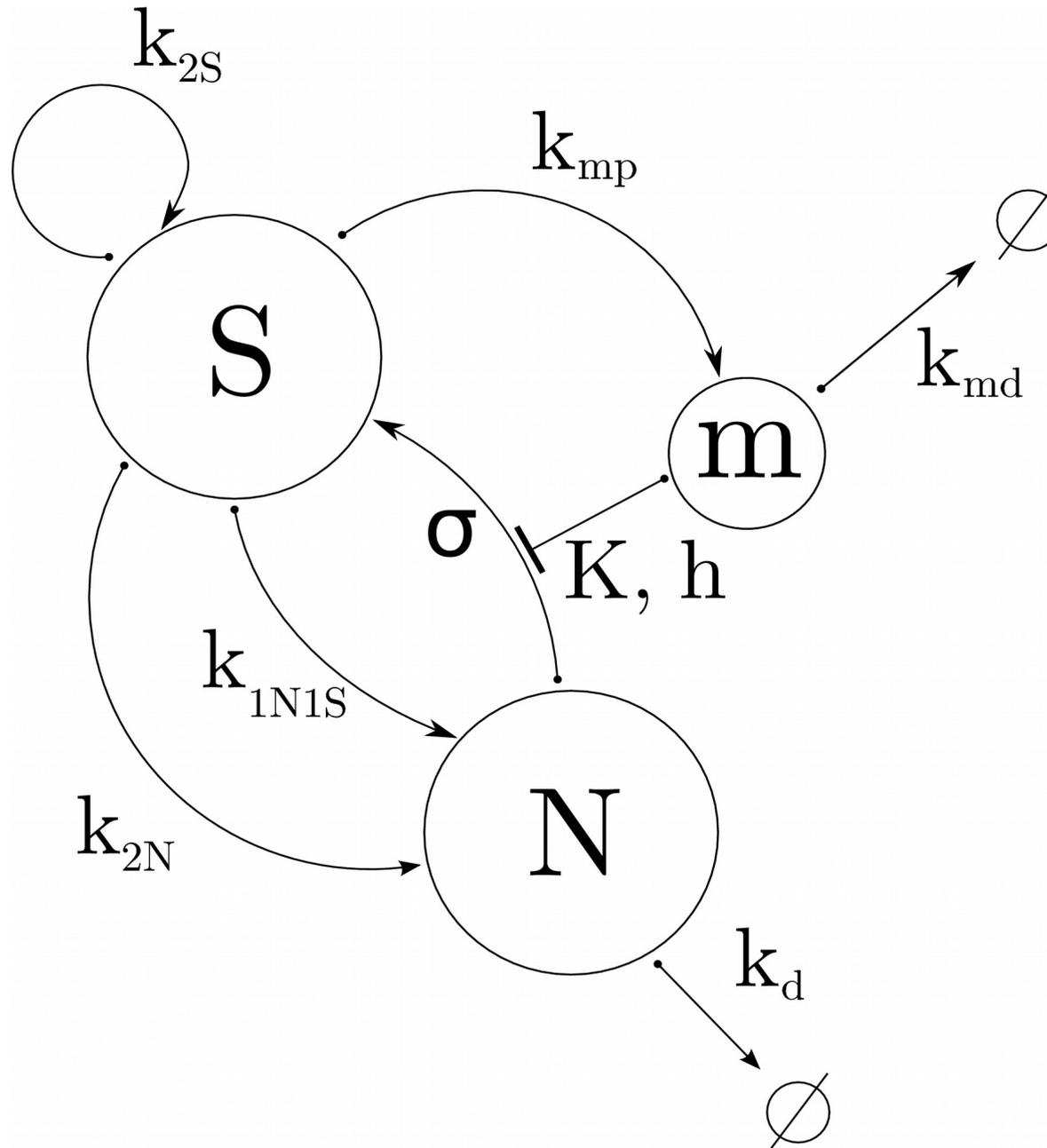
Observed Overshoot of CSC Population in a Melanoma (initially reduced under 1% of total)



Sellerio et al., Sci. Rep., 2015

The fraction of CSC grows from less than 1% of total cells to a maximum of 25% after some days and then decreases to a steady value.

Reaction Network of the model



Model 1: Rate Equations

Initial Values: $m(0) = m_0$, $S(0) = S_0$, $N(0) = N_0$.

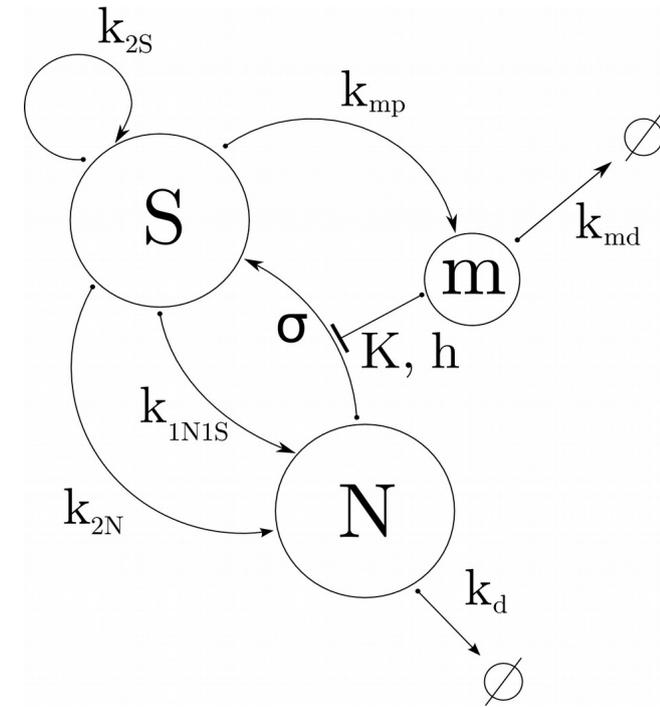
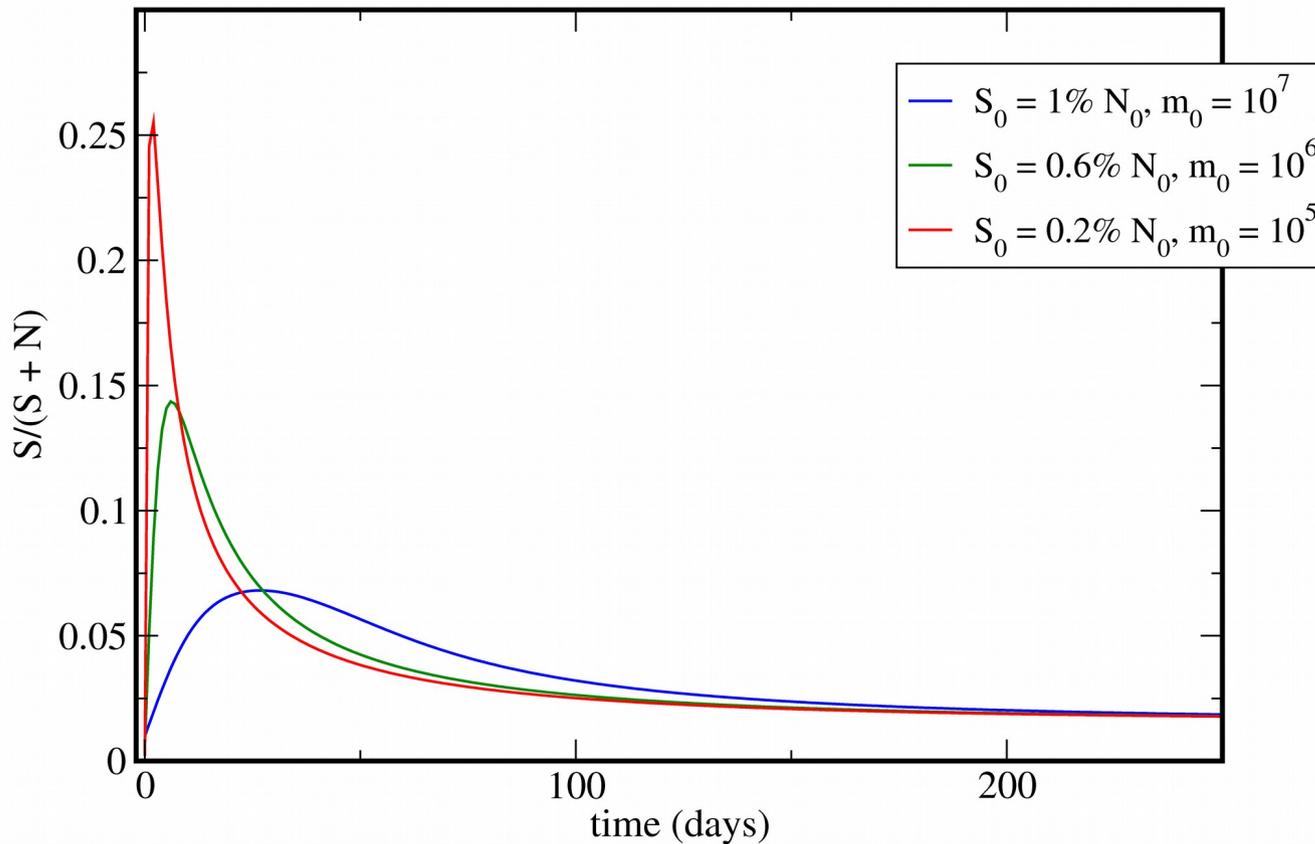
$$\begin{cases} \dot{m} = k_{np}S - k_{nm}m \\ \dot{S} = (k_{2S} - k_{2N})S + \sigma N \left(1 - \frac{m^h}{k^h + m^h} \right) \\ \dot{N} = (2k_{2N} + k_{1N1S})S - \left(k_d + \sigma \left(1 - \frac{m^h}{k^h + m^h} \right) \right) N \end{cases}$$

- Simple model;
- presence of many unknown parameters.

Model 1: Rate Eq. Output

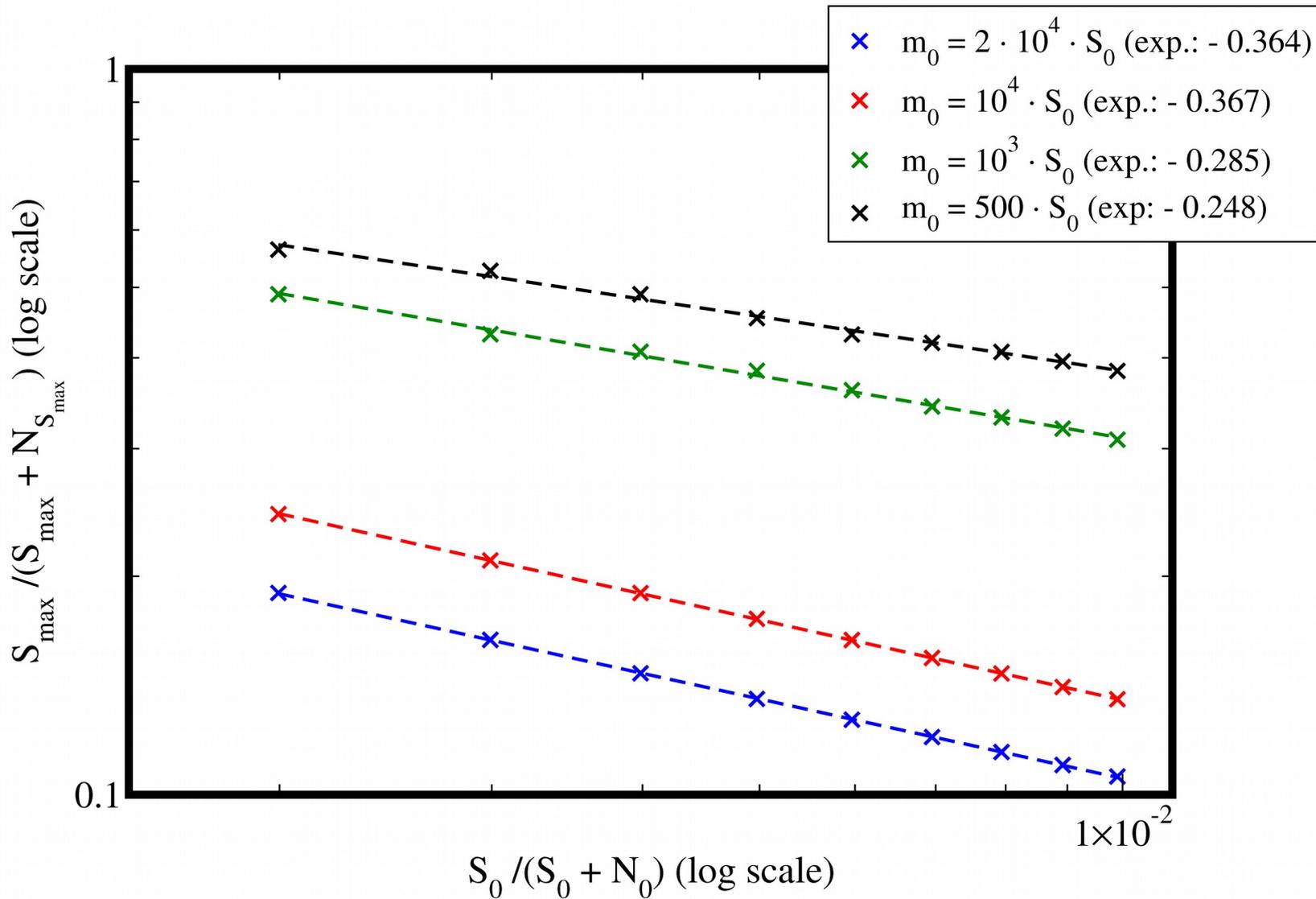
We are interested in the dynamics of $\frac{S}{S + N}$

Concentration dynamics of S cells
 $N_0 = 10^6, h = 1.$



Additional hypothesis:
 removal of S cells
 causes a removal of a
 certain amount of m

Power law for maxima of concentration of S cells with respect to initial concentration (for different ratios S_0/m_0)

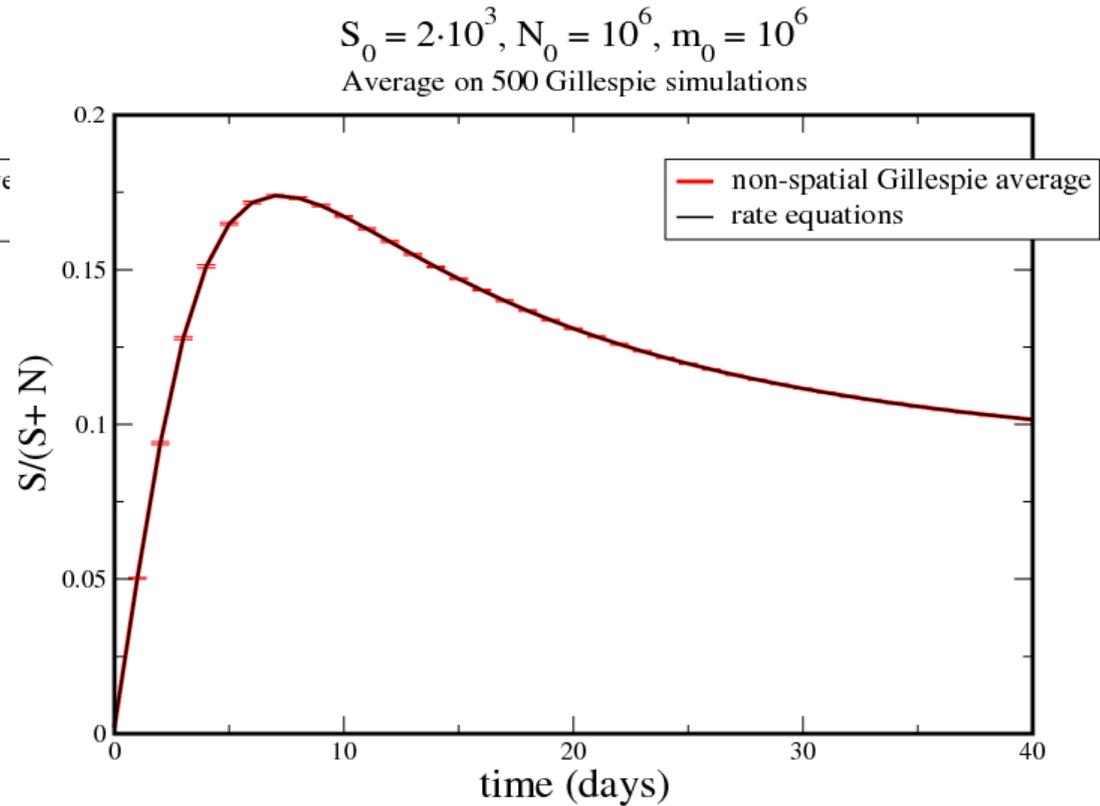
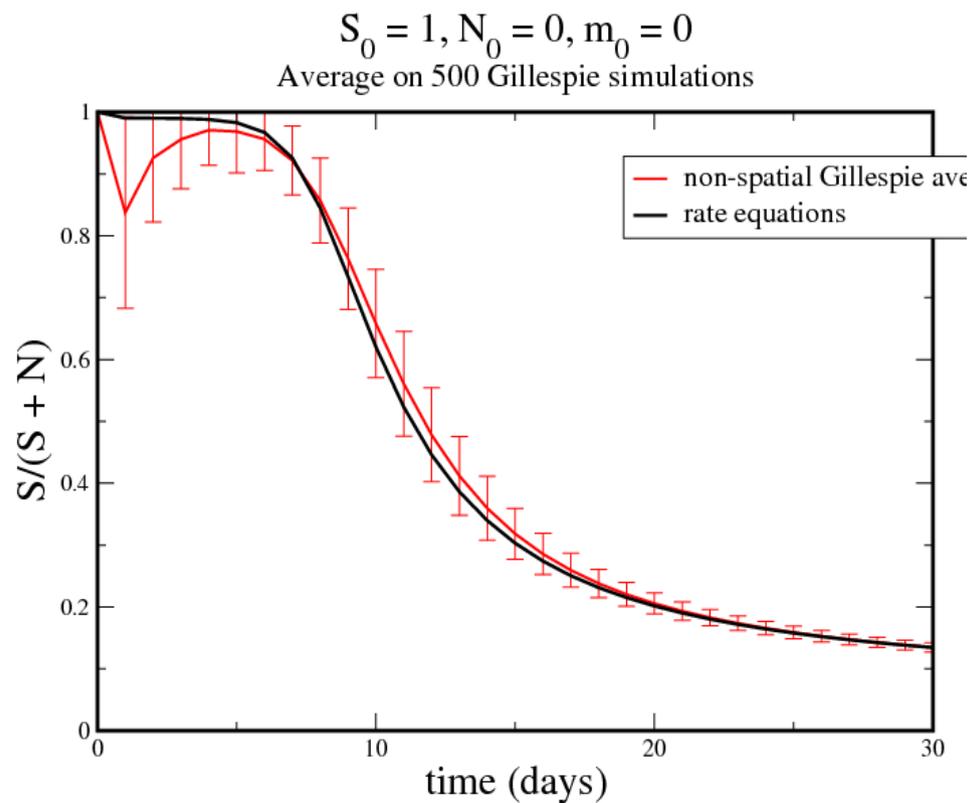


To improve our model, consider as a first step:

- the discrete nature of cells and molecules;
- possibility of reactions with low number of cells and molecules involved (**failure of well-mixed system hypothesis in most biochemical systems**);
- stochastic noise in biochemical reactions.

This lead to modeling the proposed reaction network via the **non-spatial Gillespie algorithm** (computational consistent solution of the chemical master equations).

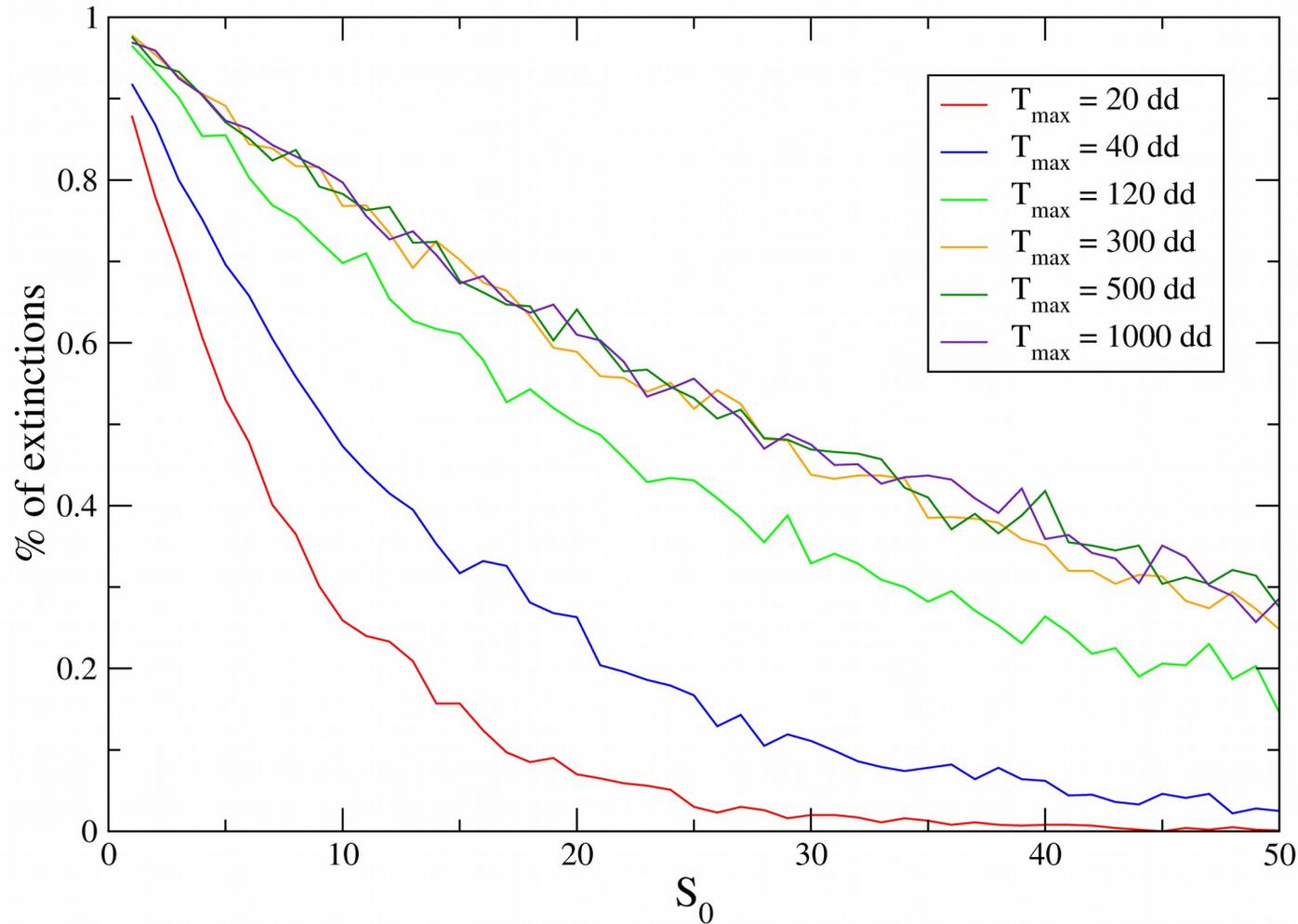
Model 2: Gillespie average vs rate equations



For high rates of spontaneous extinction of S cells dynamics, averages shall not match rate equations (intrinsic stochastic nature of the system).

Spontaneous extinction of tumor dynamics in absence of switch (small $\varepsilon = k_{2S} - k_{2N}$)

% of spontaneous extinctions in absence of switching ($\sigma = 0$)

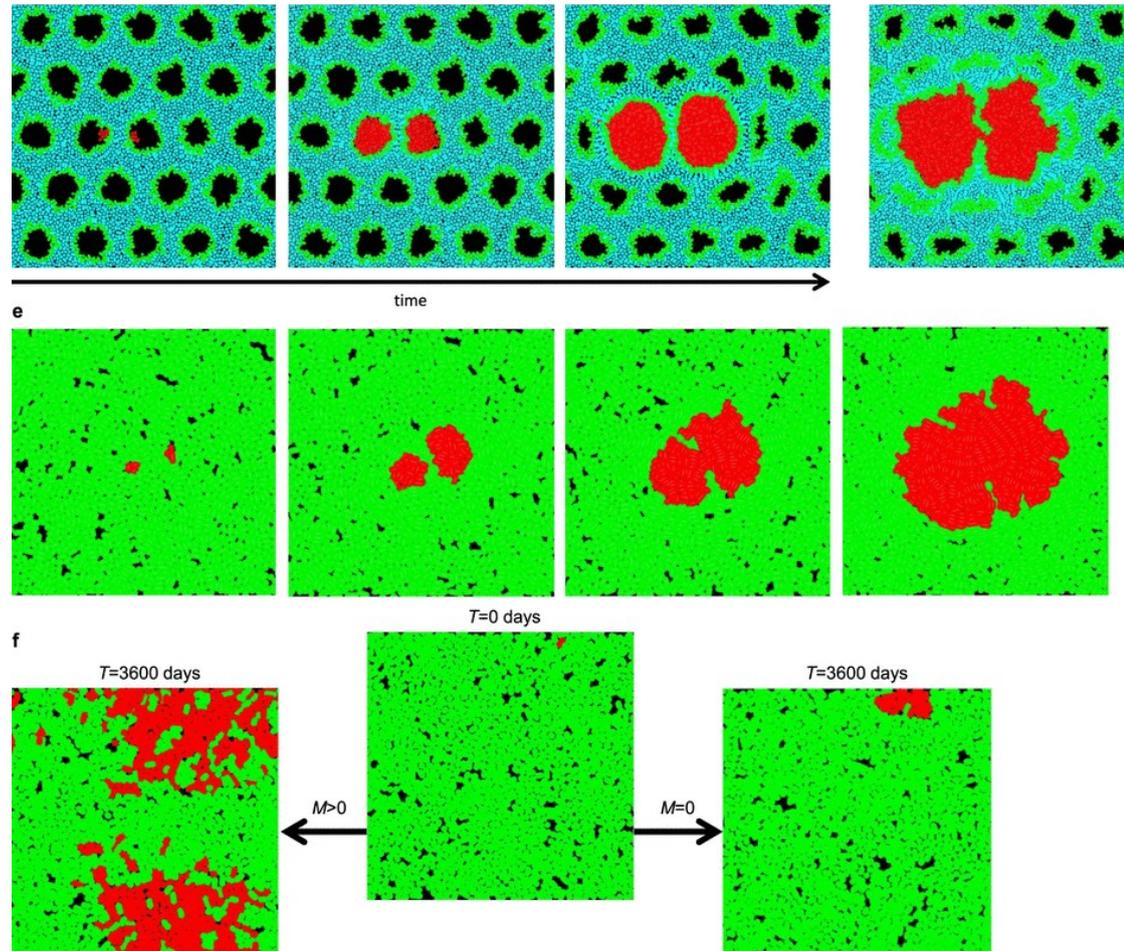


Next step: modeling in space

- Phenotypic switch dependent on local cell-molecule interactions;
- molecule m are produced in S cells sites and then diffuse;
- Effects of crowded environment on cell division rates.

Future Developments: Spatial Gillespie (2d and 3d)

- Role of diffusion of molecule m ;
- Stochastic fluctuations in space;
- Detection of spatial patterns to match images observed at the microscope.



Waclaw et al., *Nature*, 2015.