Testing the AdS/CFT correspondence

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Outline

1. General context: String Theory
2. Introduction to AdS/CFT correspondence
3. How to use and test the correspondence

Why String Theory?

Modern physics is based on quantum physics to describe atoms and particles while general relativity is used for stars, galaxies, etc...

...but stars and galaxies are ultimately made of particles: they all have to be described by the same theory!

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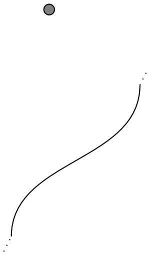
STRING THEORY

What is String Theory?

ordinary quantum field theory:

quantization of point particles

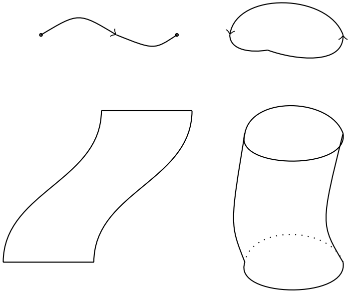


worldline

$$S = \frac{1}{2}\int d\tau(\eta^{-1}\dot{x}^{\mu}\dot{x}_{\mu} - \eta m^2)$$

string theory:

quantization of 1 dim objects



worldsheet

$$S = -\frac{1}{4\pi\alpha}\int d\tau d\sigma\sqrt{-\gamma}\gamma^{ab}\partial_a X^{\mu}\partial_b X_{\mu}$$

$$\alpha \sim l_s^2 \sim l_p^2$$

What is String Theory?

Vibration of strings $\rightarrow$ that's how particles are described in String Theory

We want a consistent relativistic quantum theory which describes bosons and fermions in curved space...

...we need to add:

- supersymmetry
- extra dimensions $\rightarrow$ 10-dim space: $M_4 \times C_6$

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String Theory spectrum

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Type IIB → particles

$$(\phi, g_{\mu\nu}, B_{\mu\nu}) + (C^0, C^2, C^4) + \text{fermions}$$

$$+ \text{massive states: } m \sim \frac{1}{l_s}$$

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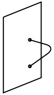
$+$ massive states: $m \sim \frac{1}{l_s}$

Dp brane [Polchinski '95]

p dimensional hyperplane:

place where open string ends



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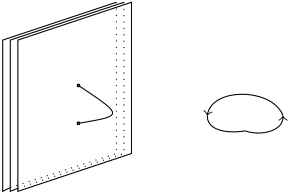
charged and massive

extended dynamic object

Origin of the correspondence

[Maldacena '97]

N coincident D3 branes in type IIB superstring theory on $M_4 \times \mathbb{R}_6$



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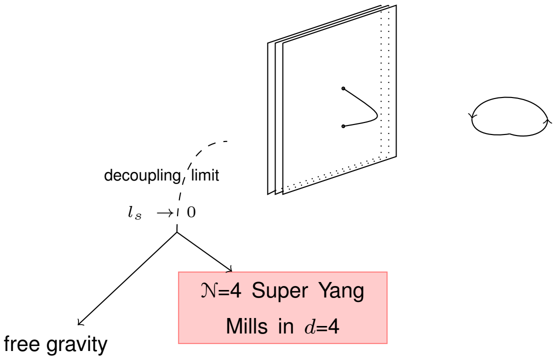

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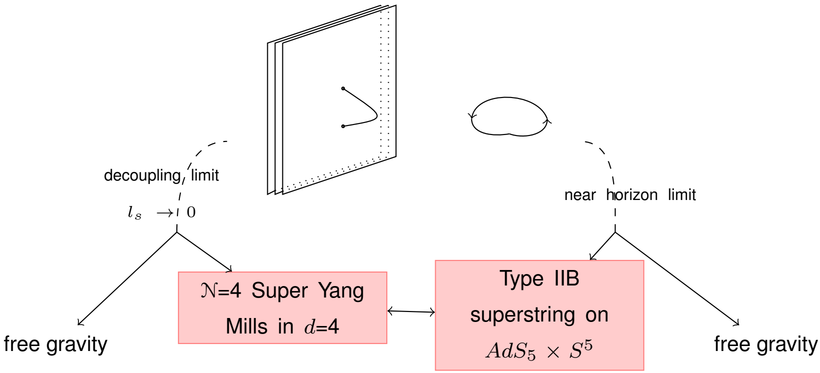

$\text{AdS}_5/\text{CFT}_4$ correspondence

Type IIB superstring on $AdS_5 \times S^5$ ⟷ $\mathcal{N}$=4 SYM $SU(N)$ in d=4

$SO(2,4) \times SO(6)$ = background isometry group

$SO(2,4) \times SO(6)_R$ = super conformal group

AdS_5/CFT_4 correspondence

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$SO(2,4) \times SO(6)$ = background isometry group		$SO(2,4) \times SO(6)_R$ = super conformal group
parameters: R, l_s, g_s		parameters: N, g_{ym}

$$\frac{R^2}{l_s} = g_{ym}\sqrt{N}$$

weak/strong duality

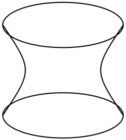
AdS/CFT correspondence

Key ingredients

AdS$_{d+1}$ = Anti de Sitter space in $d+1$ dimensions

gravity theory in Anti de Sitter space:

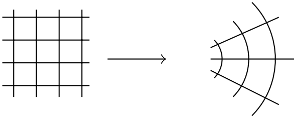
- maximally symmetric solution of Einstein eq. with $\Lambda < 0$
- isometry group $SO(2,d)$
- has boundary: d dimensional Minkowki space



CFT$_d$ = Conformal field theory in d dimensions

quantum field theory invariant under conformal group $SO(2,d)$:

- scale tranformations
- special conformal transformations

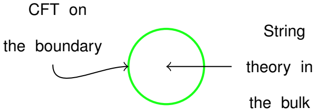


AdS/CFT correspondence

How can we use the correspondence? [Witten '98]

$$\langle \exp \int \phi_0 O \; d^4x \rangle_{CFT} = Z_s(\phi_0)$$

partition function of SYM "on the boundary" — partition function of string theory



When $l_s \to 0$ and $g_s \to 0$ string theory is approximated by classical supergravity, which correspond to a $SU(N)$ SYM with $N \to \infty$ (planar) at strong coupling:

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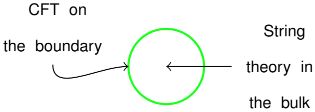

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How can we test the correspondence?

Computation of correlation functions in $\mathcal{N}=4$ SYM

Constraints on 2 and 3 point correlation functions from conformal algebra:

$$\langle O_i(x_1)O_j(x_2)\rangle = \frac{a\,\delta_{ij}}{|x_1-x_2|^{2\Delta}}$$

$$\langle O_i(x_1)O_j(x_2)O_k(x_3)\rangle = \frac{c_{ijk}}{|x_1-x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_1-x_3|^{\Delta_1+\Delta_3-\Delta_2}|x_2-x_3|^{\Delta_2+\Delta_3-\Delta_1}}$$

Chiral primary operators:

→ are protected operators: Δ doesn't receive quantum corrections

→ they correspond to supergravity states

→ matching of 2 and 3 point correlations function found at weak and strong coupling

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Non protected operators:

$\rightarrow$ they receive an anomalous dimension: $\Delta = d + \gamma(g)$

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e.g. Konishi operator: $\mathcal{K} =$ tr $\phi_k \phi_k$

They can be used for testing the correspondence:

- 2 point function: matching with strong coupling sector found for special operators
  BMN ["quasi string" states]
  long operators [via integrability]
  short operators
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Testing the correspondence

Superspace techniques

The action of $\mathcal{N}$=4 SYM:

$$S = \int d^x d^4\theta \, tr \left(e^{-gV} \bar{\Phi}_i e^{gV} \Phi^i \right) + \frac{1}{2g^2} \int d^x d^2\theta \, tr \left(W^\alpha W_\alpha \right)$$
$$+ i \frac{g}{3!} \int d^4 x d^2\theta \, \epsilon^{ijk} \, tr \left(\Phi_i \left[\Phi_j, \Phi_k \right] \right) + \text{h.c.}$$

$\mathcal{N} = 1$ super Feynman rules:

Propagators : $\langle V^a V^b \rangle = -\frac{\delta^{ab}}{p^2}$ $\langle \Phi_i^a \bar{\Phi}_j^b \rangle = \delta_{ij} \frac{\delta^{ab}}{p^2}$

Vertices : $V_C = -\frac{g}{3!} \epsilon^{ijk} f_{abc} \Phi_i^a \Phi_j^b \Phi_k^c$ $V_V^{(1)} = ig f_{abc} \delta^{ij} \bar{\Phi}_i^a V^b \Phi_j^c$

$V_V^{(2)} = \frac{g^2}{2} \delta^{ij} f_{adm} f_{bcm} V^a V^b \bar{\Phi}_i^c \Phi_j^d$

$V_V^{(3)} = \frac{ig}{2} f_{abc} V^a D^\alpha V^b \bar{D}^2 D_\alpha V^c$

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Conclusions

Calculation of correlation functions is crucial for:

- testing the AdS/CFT correspondence
- improving our knowledge of $\mathcal{N} = 4$ SYM and string theory

Future perspectives:

- calculation of the three-point correlation function at two loops in the weak coupling regime
- match with the one at strong coupling?

Thanks for your attention!