

Single-Top in t-channel at NNLO

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in collaboration with

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Outline

Top quark physics

Single-Top Production

QCD corrections to t-channel Single-Top cross-section

Identikit

m_t	$\tau_t \simeq \frac{h}{\Gamma_t}$	$\tau_{had} \simeq \frac{h}{\Lambda_{QCD}}$	$\tau_{spinflip} \simeq h \left(\frac{\Lambda_{QCD}^2}{m_t} \right)^{-1}$
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mass *theory*: convention-dependent parameter in the SM

experiments: $m_t^{exp} = 172.6 \pm 1.4 \text{ GeV}$

spin *theory*: top quark is a **spin-1/2** fermion

experiments: no dedicated experiment

- ▶ observed $t \rightarrow bW^+$ implies t is a fermion¹
- ▶ $\sigma(t\bar{t})$ implies $s_t = 1/2$

lifetime *theory*: $\tau_t \simeq 5 \times 10^{-25} \text{ s} < \tau_{had} \simeq 3 \times 10^{-24} \text{ s} < \tau_{spinflip}$

\Rightarrow **Spin** polarization and/or spin-spin correlation **visible effects**

experiments: $\Gamma_t \sim 1.3 \text{ GeV}$ smaller than resolution at both Tevatron and LHC

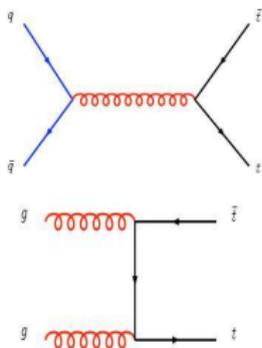
Weak charge *theory*: top + bottom $\rightarrow SU(2)_W$ doublet $\begin{pmatrix} t \\ b \end{pmatrix}$

experiments: $|V_{tb}| \gg |V_{td}|, |V_{ts}|$ but no direct measurement of $|V_{tb}|$

¹known b and W^+ spins + angular momentum conservation

Top Production

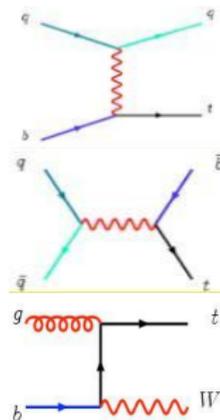
$t\bar{t}$ Pair Production



Hadronic production $\Rightarrow \mathcal{O}(\alpha_s^2)(\text{LO})$

- ▶ $\sim 10\text{pb}$ at Tevatron
- ▶ $\sim 1\text{nb}$ at LHC

Single T Production



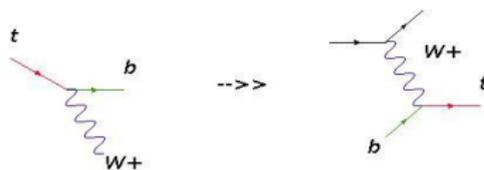
Weak production $\Rightarrow \mathcal{O}(\alpha_W^2)(\text{LO})$

- ▶ $\sim 2\text{pb}$ at Tevatron
- ▶ $\sim 300\text{pb}$ at LHC

'Weak' cross-section greater than expected... thanks to **PHASE SPACE !**

Why studying Single Top?

- ▶ $\sigma_{single\ top} \propto |V_{tb}|^2 \Rightarrow$ direct measurement of the tbW vertex

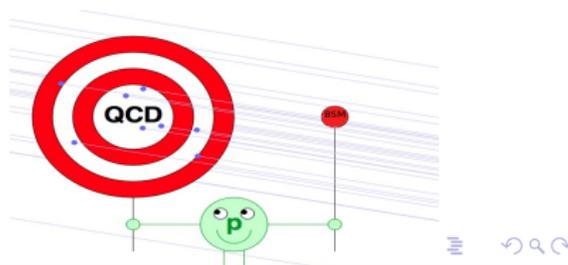


- ▶ source of polarized top quarks \Rightarrow structure of top *Charged Weak Interaction*
- ▶ possible window for BSM physics (ex. FCNC, top+charged SUSY Higgs boson, handedness of the coupling, vector (extra W) resonances)
- ▶ it represents background to some important new physics processes ($p\bar{p} \rightarrow W^+H, p\bar{p} \rightarrow H \rightarrow W^+W^-$)

We need to compute $\sigma_{t(\bar{t})}$ as precisely as we can...

The most important are QCD corrections !!!

$$\sigma_{t(\bar{t})} = \sigma_0 + \alpha_S \sigma_1 + \alpha_S^2 \sigma_2 + \dots$$



About t-channel...

In our analyses, we concentrate on t-channel...Why?

1. t-channel has the **largest cross-section** @LHC(Tevatron)!

$$\sigma_{t(\bar{t})}^{t-ch} > \sigma_{t(\bar{t})}^{s-ch}, \sigma_{t(\bar{t})}^{tW}$$

2. NLO QCD corrections are small ($\sim 5\%$), so $\sigma_{t(\bar{t})}^{t-ch}$ may become one of the most precisely predicted observables in Top Physics..
3. BSM: production of single top quarks by sizeable flavour-changing neutral currents, associated production of a top quark and charged Higgs boson in SUSY

$$\sigma_{t(\bar{t})}^{t-ch} =$$

The diagram shows the sum of tree-level and NLO QCD corrections for t-channel single top production. The tree-level process is a b quark and an anti-b quark annihilating into a top quark and an anti-top quark via a t-channel gluon exchange. The NLO corrections are shown as a series of diagrams: a gluon self-energy on the incoming b quark line, a gluon self-energy on the incoming anti-b quark line, a gluon self-energy on the t-channel gluon propagator, a gluon self-energy on the outgoing top quark line, and a gluon self-energy on the outgoing anti-top quark line. The diagrams are summed together, indicated by plus signs and an ellipsis.

t-channel: Status of the art

NLO QCD is fully explored ..

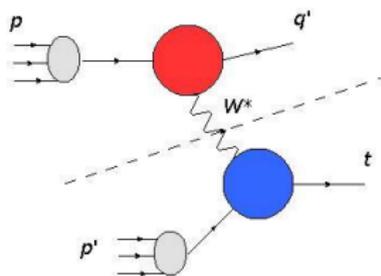
- ▶ NLO corrections ($\sim 5\%$ for LHC) for inclusive cross-section
- ▶ fully differential NLO cross-section
- ▶ NLO analyses including semi-leptonic top decays
- ▶ matching of NLO computation with parton shower MC
- ▶ EW corrections within the SM and MSSM ($\sim 1\%$)



It's time for NNLO!!



Outline of the project-Ingredients : 1.a Structure function approach



$$d\sigma \propto \frac{1}{(Q^2 + M_W^2)} MW_{\mu\nu}(x_1, Q^2, 0) \frac{1}{(Q^2 + M_W^2)} MW^{\mu\nu}(x_2, Q^2, m_t^2) dQ^2 dW_1^2 dW_2^2 \quad (1)$$

$$\begin{aligned} MW_{\mu\nu}(x, Q^2, m^2) = & F_1(x, Q^2, m^2) \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) \\ & + \frac{F_2(x, Q^2, m^2)}{P \cdot q} \left(P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) \left(P_\nu - \frac{P \cdot q}{q^2} q_\nu \right) \\ & - i \frac{F_3(x, Q^2, m^2)}{2P \cdot q} \epsilon_{\mu\nu\rho\sigma} P^\rho q^\sigma \\ & + F_4(x, Q^2, m^2) q_\mu q_\nu + F_5(x, Q^2, m^2) (P_\mu q_\nu + P_\nu q_\mu) \quad (2) \end{aligned}$$

Outline of the project - Ingredients : 1.b Master QCD formula

$$d\sigma_X = \sum_{a,b} \int_{x_1}^1 \int_{x_2}^1 \frac{dz_1}{z_1} \frac{dz_2}{z_2} f_a\left(\frac{x_1}{z_1}, \mu_F^2\right) f_b\left(\frac{x_2}{z_2}, \mu_F^2\right) \times d\hat{\sigma}_{ab \rightarrow X}(z_1, z_2, \alpha_s(\mu_R^2), \frac{Q^2}{\mu_F^2} \frac{Q^2}{\mu_R^2}) \quad (3)$$

1. $f_a(x_1, \mu_F^2)$, $f_b(x_2, \mu_F^2)$ are **Parton Distribution Functions** (from experimental data, but evolution from the theory)
2. $d\hat{\sigma}_{ab \rightarrow X}$ is the **Coefficient Function**, namely the partonic cross-section, describing the short distance interaction (from theory, it can be computed as expansion in α_s)

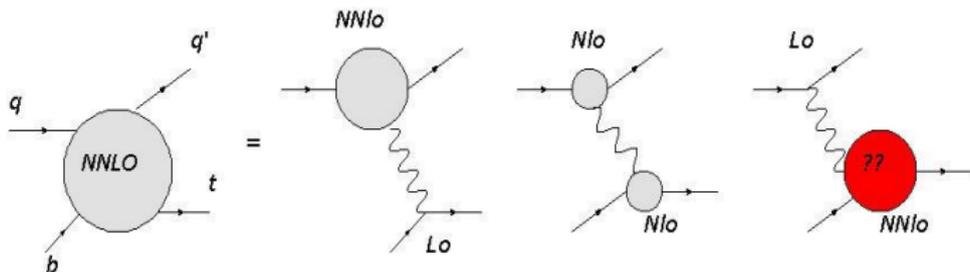
Combining eq.(1),(2) and (3), we obtain for $F_i(x, Q^2, m^2)$:

$$F_i(x, Q^2, m^2) = \sum_q \int_x^1 \frac{dz}{z} f_q\left(\frac{x}{z}, \mu_F^2\right) \times d\hat{\sigma}_{qV^* \rightarrow X}(z, \alpha_s(\mu_R^2), \frac{Q^2}{\mu_F^2} \frac{Q^2}{\mu_R^2}) \quad (4)$$

Outline of the project - Ingredients : 2.a Decomposition of $\mathcal{O}(\alpha_s^2)$ contribution $\hat{\sigma}^{(2)}$ to $\hat{\sigma}_{t(\bar{t})}^{t-ch}$ cross-section

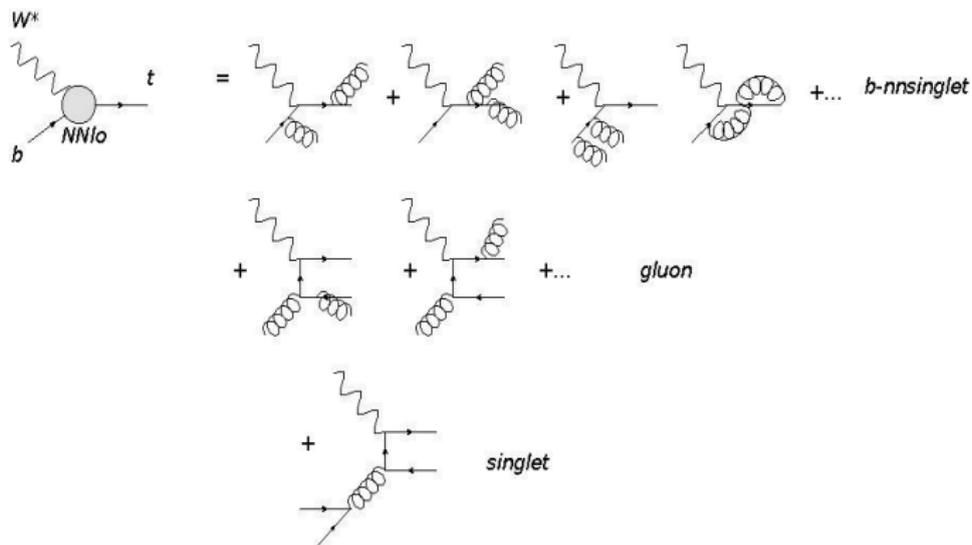
$F^{(2)}(x, Q^2, m_t^2)$ is the only piece left to complete the puzzle...

Indeed, at partonic level $d\hat{\sigma}_{qW^* \rightarrow t+X}^{(2)}$ is the only unknown part of NNLO coefficient function.



$$\begin{aligned}
 d\hat{\sigma}^{(2)} &= d\hat{\sigma}(q \rightarrow q' W^*)^{(2)} \times d\hat{\sigma}(bW^* \rightarrow t)^{(0)} \\
 &+ d\hat{\sigma}(q \rightarrow q' W^*)^{(1)} \times d\hat{\sigma}(bW^* \rightarrow t)^{(1)} \\
 &+ d\hat{\sigma}(q \rightarrow q' W^*)^{(0)} \times d\hat{\sigma}(bW^* \rightarrow t)^{(2)}
 \end{aligned} \tag{5}$$

Outline of the project - Ingredients : 2.b Decomposition of $\mathcal{O}(\alpha_s^2)$ contribution $F_i^{(2)}(x, Q^2, m_t^2)$ to massive structure function $F_i(x, Q^2, m_t^2)$



$$F^{(2)}(x, Q^2, m_t^2) = F_q^{(2)}(x, Q^2, m_t^2) + F_g^{(2)}(x, Q^2, m_t^2) + F_s^{(2)}(x, Q^2, m_t^2)$$

Outline of the project-Ingredients : 3. Master Formula

1. $F_g^{(2)}(x, Q^2, m_t^2) + F_s^{(2)}(x, Q^2, m_t^2)$ can be extracted numerically from MC
2. $F_q^{(2)}(x, Q^2, m_t^2)$ is the real object of our guess!! It is known in 2 kinematical regions..

$$Q^2 \gg m_t^2 \Rightarrow F(x, Q^2, 0)$$

$$x \rightarrow 1 \Rightarrow F_{\text{soft}}(x, Q^2, m_t^2)$$

$$F_q^{(k)}(x, Q^2, m_t^2) = F_{q,\text{soft}}^{(k)}(x, Q^2, m_t^2) + F_q^{(k)}(x, Q^2, 0) - F_{q,\text{soft}}^{(k)}(x, Q^2, 0) + R_q^{(k)}(x, Q^2, m_t)$$

R_q is the *finite part*, with the requirements:

$$\text{massless } R_q^{(k)}(x, Q^2, m_t^2) \xrightarrow{m_t \rightarrow 0} 0$$

$$\text{soft } R_q^{(k)}(x, Q^2, m_t^2) \xrightarrow{x \rightarrow 1} a(m_t) + (1-x)b(m_t) + \dots$$

If R_q is 'small'... \Rightarrow our guess

$F_{q,\text{approx}}^{(k)}(x, Q^2, m_t^2) = F_{q,\text{soft}}^{(k)}(x, Q^2, m_t^2) + F_q^{(k)}(x, Q^2, 0) - F_{q,\text{soft}}^{(k)}(x, Q^2, 0)$ is good!!

Outline of the project - NLO preliminary analyses

@NLO : full massive computation is still accessible...

1. checked $C^{(1)}(x, Q^2, m_t^2)$ results in literature by redoing full NLO computation for $d\hat{\sigma}_{qW^* \rightarrow t+X}$
2. implemented these results in a standalone MC code, to perform numerical phase space integration and convolution with PDFs, thus checking NLO results for $F^{(1)}(x, Q^2, m_t^2)$
3. performed analytically massless and soft limits on $C_q^{(1)}(x, Q^2, m_t^2)$
4. implemented this limits in our MC, to get $F_{q,soft}^{(1)}(x, Q^2, m_t^2)$, $F_q^{(1)}(x, Q^2, 0)$, $F_{q,soft}^{(1)}(x, Q^2, 0)$

$$R^{(1)}(x, Q^2, m_t^2) = F^{(1)}(x, Q^2, m_t^2) - F_{soft}^{(1)}(x, Q^2, m_t^2) - F^{(1)}(x, Q^2, 0) + F_{soft}^{(1)}(x, Q^2, 0)$$

... everything is known, so we can estimate $\mathcal{O}(R^{(1)})$...

@physical Top mass

m_t	F_{m_t}	$F_{m_t,a}$	R	R/F_{m_t}
172.	41.848819	41.8501376	-0.0013186	-0.000031508655 < MC error

Outline of the project - NNLO.. work in progress!

$$\begin{aligned}
 F_{q,approx}^{(2)}(x, Q^2, m_t^2) &= F_{q,soft}^{(2)}(x, Q^2, m_t^2) + F_q^{(2)}(x, Q^2, 0) - F_{q,soft}^{(2)}(x, Q^2, 0) \\
 F_{approx}^{(2)}(x, Q^2, m_t^2) &= F_{q,approx}^{(2)}(x, Q^2, m_t^2) + F_g^{(2)}(x, Q^2, m_t^2) + F_s^{(2)}(x, Q^2, m_t^2)
 \end{aligned}$$

1. is $C^{(2)}(x, Q^2, 0)$ is known in literature \rightarrow massless approximation
 $F_q^{(2)}(x, Q^2, 0)$ is straight obtained with our MC
2. $F_g^{(2)}(x, Q^2, 0) + F_s^{(2)}(x, Q^2, 0)$ to be extracted numerically from existing MC
3. $F_{soft}^{(2)}(x, Q^2, m_t^2)$ and $F_{soft}^{(2)}(x, Q^2, 0)$ have to be taken from *soft resummation*...

Soft resummation @NNLO

$$\begin{aligned}
\frac{d^2\hat{\sigma}^{(2)}}{dt du} = & F^B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \left\{ \frac{1}{2} c_3^2 \left[\frac{\ln^3(s_4/m_t^2)}{s_4} \right]_+ + \left[\frac{3}{2} c_3 c_2 - \frac{\beta_0}{4} c_3 + C_F \frac{\beta_0}{8} \right] \left[\frac{\ln^2(s_4/m_t^2)}{s_4} \right]_+ \right. \\
& + \left[c_3 c_1 + c_2^2 - \zeta_2 c_3^2 - \frac{\beta_0}{2} T_2 + \frac{\beta_0}{4} c_3 \ln \left(\frac{\mu_R^2}{m_t^2} \right) + \frac{3}{2} C_F K - \frac{3}{16} C_F \beta_0 + 4\Gamma_{S12}^{(1)} \Gamma_{S21}^{(1)} \right] \left[\frac{\ln(s_4/m_t^2)}{s_4} \right]_+ \\
& + \left[c_2 c_1 - \zeta_2 c_3 c_2 + \zeta_3 c_3^2 + \frac{\beta_0}{4} c_2 \ln \left(\frac{\mu_R^2}{s} \right) - \frac{\beta_0}{2} C_F \ln^2 \left(\frac{m_t^2 - t}{m_t^2} \right) - \frac{\beta_0}{2} C_F \ln^2 \left(\frac{m_t^2 - u}{m_t^2} \right) \right. \\
& - C_F K \ln \left(\frac{(m_t^2 - u)(m_t^2 - t)}{m_t^4} \right) + B^{(2)} + 3D^{(2)} + C_F \frac{\beta_0}{4} \ln^2 \left(\frac{\mu_F^2}{s} \right) - C_F K \ln \left(\frac{\mu_F^2}{s} \right) \\
& \left. + \frac{3\beta_0}{8} C_F \ln^2 \left(\frac{m_t^2}{s} \right) - C_F \left(\frac{K}{2} - \frac{3}{16} \beta_0 \right) \ln \left(\frac{m_t^2}{s} \right) + 2\Gamma_{S11}^{(2)} + \left(4\Gamma_{S12}^{(1)} \Gamma_{S21}^{(1)} + 4(\Gamma_{S11}^{(1)})^2 \right) \ln \left(\frac{m_t^2}{s} \right) \right] \left[\frac{1}{s_4} \right]_+ \left. \right\}
\end{aligned}$$

just a matter of disentanglement ...



Conclusion

- ▶ Top Physics
- ▶ Single Top Production
- ▶ QCD corrections for Single Top Production in t-channel
 1. Ingredients
 2. NLO preliminary analyses... our guess looks promising!
 3. NNLO strategy

NNLO first result for CHARGED DIS structure functions $F^{(2)}(x, Q^2, m_t^2)$ in a couple of month!

..thank you for your attention