Modern Approaches to Scattering Amplitudes

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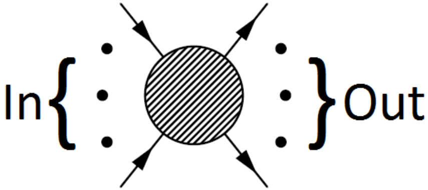
UNIVERSITÀ DEGLI STUDI DI MILANO
FACOLTÀ DI SCIENZE E TECNOLOGIE

Outline

Scattering Of Particles



Cross Sections and Rates $\longrightarrow$

$$\frac{d\sigma}{d\Omega} = (2\pi)^4 m_i m_f \frac{p_f}{p_i} |\mathcal{A}_{i,f}|^2$$

$$\mathcal{A}_{i,f} = \langle \alpha_{in} | \beta_{out} \rangle \qquad \alpha, \beta = (p^{\mu}, h, c)$$

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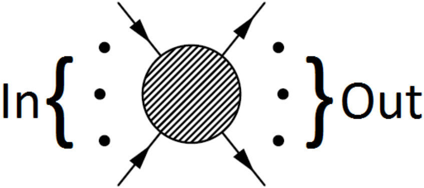

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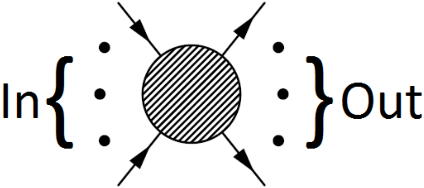

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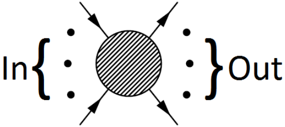

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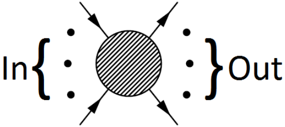

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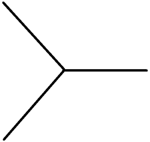
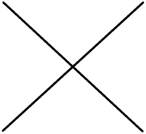
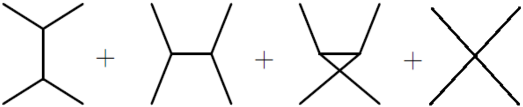
Feynman Diagrams

Lagrangian

$$\mathrm{L} = \int d^4x \mathcal{L}(\phi, \partial_\mu \phi), \quad \text{e.g. } \mathrm{L} = \int d^4x \; \partial_\mu \phi \partial^\mu \phi + g_3 \phi^3 + g_4 \phi^4 + \dots$$

$g_3 \rightarrow$  $g_4 \rightarrow$ 

$A_4 =$  $+ \dots$

Cumbersome computation!

Feynman Expansion

$$A_n = \sum_{\Gamma \in Graphs} Value(\Gamma)$$

- IR and UV infinities: need to renormalize
- Factorial growth of the number of diagrams $\sim n!$

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Unnecessarily Cumbersome Computation?

VOLUME 56, NUMBER 23 PHYSICAL REVIEW LETTERS 9 JUNE 1986

Amplitude for *n*-Gluon Scattering

Stephen J. Parke and T. R. Taylor
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
(Received 17 March 1986)

A nontrivial squared helicity amplitude is given for the scattering of an arbitrary number of gluons to lowest order in the coupling constant and to leading order in the number of colors.

PACS numbers: 12.38.Bx

$$A[g_1^- g_2^- \to g_3^+ g_4^+ \dots g_n^+] = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\dots\langle n1\rangle}$$

$$\langle ij\rangle \sim (p_i + p_j)^2$$

Particles and Fields

- Particles: Experimentally accesible quantum states, well defined Mass, Charges and Free Propagation
- Fields: Tools used to construct Lorentz Invariant dynamics:
  $\phi(x) = \int d^3\overrightarrow{p}\ exp(ip^\mu x_\mu) u(\overrightarrow{p})\hat{a}(\overrightarrow{p})$
  $\rightarrow\ \hat{U}(\Lambda)(\phi(x)) = S(\Lambda)(\phi(\Lambda^{-1}x)), \quad \Lambda \in SO(1,3)$
  $\rightarrow\ \mathrm{L} = \int d^4x\ \mathcal{L}(\partial_\mu\phi, \phi)$ is invariant.
- Problem: No massless vector field (e.g. A^μ in QED)
  $\hat{U}(\Lambda)(\mathrm{A}^\mu(x)) = \Lambda^\mu_\nu \mathrm{A}^\nu(\Lambda^{-1}x) + \partial^\mu\chi$
- Solution: Coupling with a conserved current J^μ
  $\mathrm{L} \rightarrow \mathrm{L} + \int J^\mu\partial_\mu\chi = \mathrm{L} - \int \partial_\mu J^\mu\chi = \mathrm{L}$

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Gauge redundancy

- Particles know nothing about gauge freedom $\rightarrow$ Scattering Amplitudes are gauge invariant (Ward Identities $p_\mu A^\mu = 0$)
- Fields transform with the gauge $\rightarrow$ Feynman diagrams are not gauge invariant

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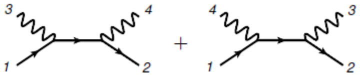
QED Compton Scattering

$iA_4(\bar{f} f \gamma\gamma) =$ 

$$A_4(\bar{f}^+ f^- \gamma^+ \gamma^-) = 2e^2 \frac{\langle 24\rangle [q_4|(-|1]\langle 1| - |3]\langle 3|)|q_3\rangle [31]}{\langle 13\rangle [13] \langle q_3 3\rangle [q_4 4]} +$$

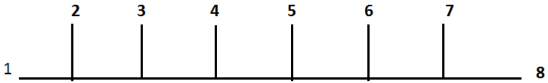
$$+ 2e^2 \frac{\langle 2q_3\rangle [3|(-|1]\langle 1| - |4]\langle 4|)|4\rangle [q_4 1]}{\langle 14\rangle [14] \langle q_3 3\rangle [q_4 4]}$$

$$= 2e^2 \frac{\langle 24\rangle}{\langle 13\rangle \langle 23\rangle}$$

Amplitudes as building blocks for Amplitudes

- Gauge Invariant Expansion? $A_n = \sum_{gauge\ \ invariants} GI$
- Amplitudes are natural invariant objects
- Feynman diagrams suggest a way: Look at poles and residues of Amplitudes

Poles and Residues of Tree Amplitudes I



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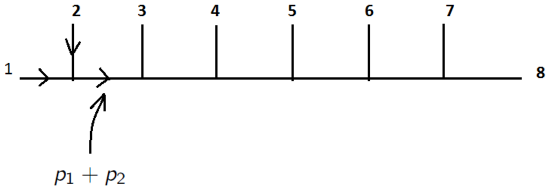

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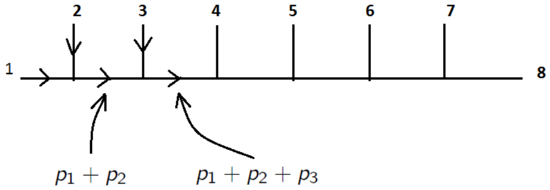

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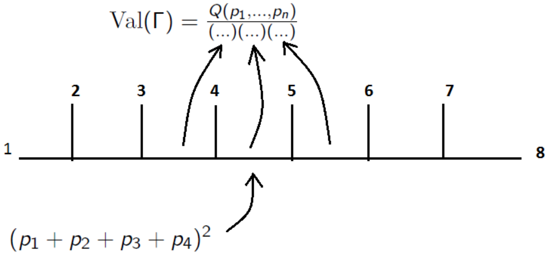
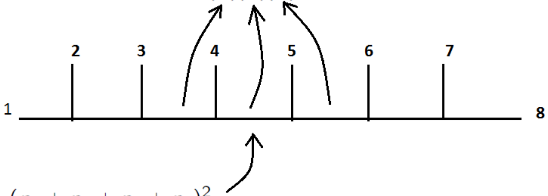
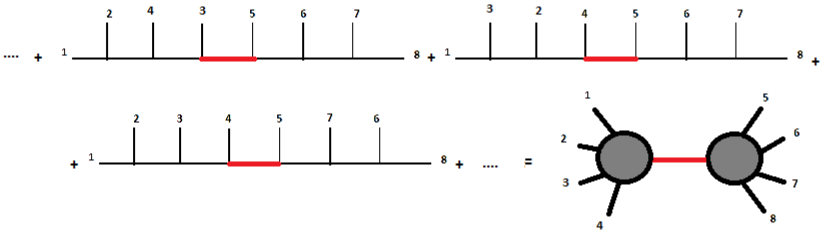


Poles and Residues of Tree Amplitudes I

$$\mathrm{Val}(\Gamma) = \frac{Q(p_1,\ldots,p_n)}{(\ldots)(\ldots)(\ldots)}$$



$$(p_1 + p_2 + p_3 + p_4)^2$$



Poles and Residues of Tree Amplitudes II

To sum up:

- $A_n(p)$ has a pole if $P_I^2 = 0$, $P_I = \sum_{i \in I} p_i$.
- The residue is given by amplitudes with fewer legs: $\mathrm{Res}\ A_n(p) = A_{n_L} A_{n_R}, \quad n_L + n_R = n + 2$.
- Tempting Conjecture: $A_n = \sum_I \frac{A_{n_L} A_{n_R}}{P_I^2}$
  Not so easy: in general one gets spurious poles
- A more refined strategy: Britto-Cachazo-Feng-Witten recursion relations.

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BCFW Recursion Relations in a Nutshell

Goal

Exploit knowledge of singularity structure in a constructive way

- Deform momenta $p_i(z) = p_i + zr_i$, $z \in \mathbb{C}$, in such a way that
  1. $p_i(z)$ are on-shell, i.e. $p_i(z)^2 = 0$
  2. Satisfy momentum conservation $\sum_i p_i(z) = 0$
  3. Mandelstam invariants are linear in z: $P_I^2(z) = P_I^2 + zD_I$
- Poles of $\frac{A_n(z)}{z}$:

$$z = 0 \quad \xrightarrow[\text{Res}]{} A_n(0) = A_n$$

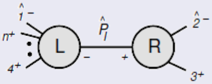
$$z_I = -\frac{P_I^2}{D_I} \xrightarrow[\text{Res}]{} \frac{A_n}{z_I} = \frac{A_{n_L}(z_I)A_{n_R}(z_I)}{P_I^2}$$

- Use Cauchy theorem in $\mathbb{CP}^1$: $A_n = \sum \frac{A_{n_L}(z_I)A_{n_R}(z_I)}{P_I^2}$

BCFW in Action

Parke-Taylor formula

$$A_n[1^-2^-3^+\dots n^+] \; = $$



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Amplitudes beyond "Maximally Helicity Violating" order

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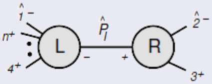

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Beyond Tree Level

- BCFW for the integrand
  $\sum_{Graphs} \int d^4 l \to \int d^4 l \sum_{Graphs}$
- Unitarity Cuts
  $A_{n,1} =$
  $\sum_i d_i(\text{box}) + \sum_i c_i(\text{triangle}) + sum_i b_i(\text{bubble}) + \text{rational}$
- The Symbol, Cluster Algebras

- QCD loop computations (Dixon)
- Unexpected UV finiteness of $\mathcal{N} = 8$ SUGRA (Bern, Carrasco, Johansson)

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Two Puzzles from BCFW

- Spurious Poles: Individual terms have spurious poles that cancels in the sum.
- Non trivial relations: Different deformation schemes $\rightarrow$ relations among Gauge invariant objects.

The simplest example:
6-point NMHV amplitude in $\mathcal{N} = 4$ Super Yang-Mills

$$\begin{aligned} A_6[1,2,3,4,5,6] &= [3,1,6,5,4] + [3,2,1,6,5] + [3,2,1,5,4] \\ &= [2,3,4,6,1] + [2,3,4,5,6] + [2,4,5,6,1] \end{aligned}$$

Kinematics in (Bosonized) Twistor Space

Kinematical Space	Space-Time	Bosonized Twistor Space
Variables	$p_i \in \mathbb{R}^{1,3}$	$Z_i \in \mathbb{CP}^4$, $\mathbf{0} = (0,0,0,0,1)$
Physical Poles	$(\sum_{a=i}^{j} p_a)^2 = 0$	$\langle 0, i, i+1, j, j+1 \rangle = 0$

$$[i,j,k,l,m] := \frac{1}{4!}\int d^4\psi \frac{\langle i,j,k,l,m\rangle^4}{\langle 0,i,j,k,l\rangle\langle 0,j,k,l,m\rangle\langle 0,k,l,m,i\rangle\langle 0,l,m,i,j\rangle\langle 0,m,i,j,k\rangle}$$

$$A_6[1,2,3,4,5,6] = [2,3,4,6,1] + [2,3,4,5,6] + [2,4,5,6,1]$$

$$[2,3,4,6,1] \to \langle 0,4,6,1,2\rangle,\ \langle 0,2,3,4,6\rangle$$

The Integrand is the volume of a 4-simplex in $\mathbb{CP}^4$, with co-dimension 1 boundaries $Z_i \cdot W = 0$! (Hodges)

The Amplitude is the volume of a *polytope*.

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Triangles and Polytopes in the Plane

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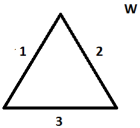

$$Area(T) = \frac{1}{2} \frac{\langle 1,2,3 \rangle}{\langle 0,1,2 \rangle \langle 0,2,3 \rangle \langle 0,3,1 \rangle} =: [1, 2, 3]$$

$$[4, 1, 3] - [2, 1, 3] = Area(P) = [2, 4, 3] - [1, 2, 4]$$

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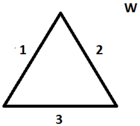

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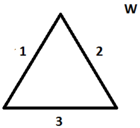

$$Area(T) = \frac{1}{2} \frac{\langle 1,2,3 \rangle}{\langle 0,1,2 \rangle \langle 0,2,3 \rangle \langle 0,3,1 \rangle} =: [1,2,3]$$

$$[4,1,3] - [2,1,3] = Area(P) = [2,4,3] - [1,2,4]$$

Triangles and Polytopes in the Plane

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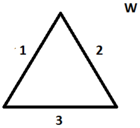

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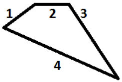

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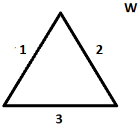

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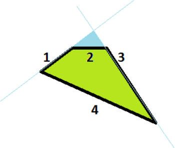

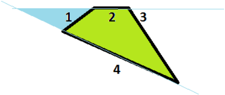

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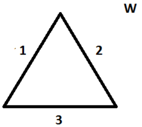

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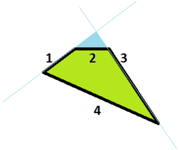

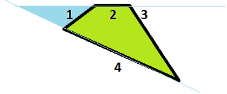

$$[4,1,3] - [2,1,3] = Area(P) = [2,4,3] - [1,2,4]$$

6-Points NMHV as a Volume

Back to higher dimension

- $A_6^{NMHV} = Volume(P)$
  $P =$
  $\{(1,2,3,4);(1,2,3,6);(1,3,4,6);(1,4,5,2);(1,4,5,6);(1,2,5,6)\}$
- P is triangulated by 3 4-simplices, non-local vertices cancels.
- Different BCFW representations give different triangulations of P

Beyond NMHV amplitudes?

The Amplituhedron

The Amplituhedron

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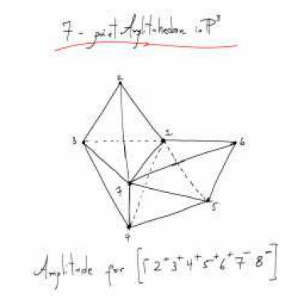
Abstract: Perturbative scattering amplitudes in gauge theories have remarkable simplicity and hidden infinite dimensional symmetries that are completely obscured in the conventional formulation of field theory using Feynman diagrams. This suggests the existence of a new understanding for scattering amplitudes where locality and unitarity do not play a central role but are derived consequences from a different starting point. In this note we provide such an understanding for $\mathcal{N} = 4$ SYM scattering amplitudes in the planar limit, which we identify as "the volume" of a new mathematical object–the Amplituhedron–generalizing the positive Grassmannian. Locality and unitarity emerge hand-in-hand from positive geometry.



$$A_n \sim \mathrm{Vol}(\mathcal{A}_n)$$

$$Gr^+(4, 4+k) \supset \mathcal{A} = C \cdot Z, \quad C \in Gr^+(4, n), \quad Z \in Gr^+(n, 4+k)$$

Summary

- Scattering Amplitudes are among the most important quantities in theoretical physics.
- Efficient way to compute them have immediate applications to experimental physics
- In a virtuous circle, these methods shred new lights upon hidden structures (and hopefully principles) in QFT

Thanks for your attention!