$\mathcal{N} = 2$ supergravity in 4 dimensions

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Supersymmetry:

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- unification of bosons (forces) and fermions (matter);
- better high energy behaviour;
- unification of gauge couplings preciser;
- provides candidates for cold dark matter.

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		$M_{\mu\nu}$ (Lorentz group)
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superalgebra:	$[B, B] = B$	(usual Poincaré)
	$[B, F] = F$	$[M, Q] \propto Q$
	$\{F, F\} = B$	$\{Q, Q\} \propto P$

Supersymmetry: field content

If $\mathcal{N}$ and λ (max value of the spin) are fixed, the field content of the theory is known.

Example: if $\mathcal{N} = 2$ and $\lambda = 2$ $\Rightarrow$ $g_{\mu\nu}$ graviton with spin 2

$\Psi_{\mu\alpha}$ gravitino with spin $\frac{3}{2}$

A_μ vector field with spin 1

One multiplet including both fermions and bosons.

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Supersymmetric version of pure general relativity

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- supersymmetric $\Rightarrow$ based on symmetry between bosons and fermions;
- based on general relativity;
- pure $\Rightarrow$ no matter fields;

BUT

- can be coupled to matter fields $\Rightarrow$ field theory describing both gravity and the other forces;

Supergravity: how

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A gauge symmetry between bosons and fermions can only be implemented in field theory if spacetime is curved (gravity).

Supergravity: why

- all the reasons for supersymmetry;
- unification of gravity and other forces;
- better than general relativity as quantum theory at high energies;
- provides scalar candidates for inflatons;
- considered the low energy limit of string theory.

$\mathcal{N} = 2$ supergravity: field content

Particles are organized in multiplets:

- supergravity multiplet $\rightarrow$

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- matter multiplets
  - n_V vector multiplets $\rightarrow$

A^α_μ	n_V vector fields
z^α	n_V scalar fields
$\chi^{\mathrm{i}\alpha}$	n_V fermions, gauginos

  - n_H hypermultiplets $\rightarrow$

q^u	$4n_H$ scalar fields, hyperscalars
ξ^A	$2n_H$ fermions, hyperinos

$\mathcal{N}=2$ supergravity: general features

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- two supercharges Q;
- additional symmetry $\Rightarrow$ the scalars z^α and q^u of the matter multiplets can be viewed as coordinates of peculiar manifolds;
- function F (prepotential) to determine all the relevant quantities in the bosonic Lagrangian;
- part of the additional symmetry can be gauged.

$\mathcal{N}=2$ supergravity: our choice

- $n_H = 0$, no hypermultiplets;
- $n_V = 3$, 3 vector multiplets;
- the 3 scalars z^α as coordinates of a manifold;
- specific choice of F, coming from quantum corrections to string theories;
- part of the additional symmetry is gauged.

$\mathcal{N}=2$ supergravity: solutions

- bosonic Lagrangian $\Rightarrow$ classical solutions
  - configurations of the bosonic fields only which satisfy the equations of motion, when all the fermionic fields vanish.

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- bosonic Lagrangian $\Rightarrow$ classical solutions
  - configurations of the bosonic fields only which satisfy the equations of motion, when all the fermionic fields vanish.
- black hole solutions:
  - rich set of geometries;
  - electric and magnetic charges;
  - entropy.

$\mathcal{N}$=2 supergravity: some details

- from the function F and the gauging;

$$e^{-1}\mathcal{L}_{bos} = \frac{1}{16\pi G} R + \frac{1}{4} \mathcal{I}_{IJ} F^I_{\mu\nu} F^{J\mu\nu} - \frac{1}{8} \mathcal{R}_{IJ} e^{-1} \epsilon^{\mu\nu\rho\sigma} F^I_{\mu\nu} F^J_{\rho\sigma}$$

$$- g_{\alpha\overline{\beta}} \partial_\mu z^\alpha \partial^\mu \overline{z}^{\overline{\beta}} - V$$

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- Ansatz for the black hole metric;

$$ds^2 = -e^{2U(r)}dt^2 + e^{-2U(r)}\left(dr^2 + e^{2\psi(r)}d\Omega^2\right)$$

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- equations of motion $\Rightarrow$ system of differential equations in U, ψ, z^{α}, to be solved.

Conclusions:

- symmetry is a powerful tool;
- introduction of supersymmetry and supergravity;
- many valuable properties;
- unification of gravity and other forces;
- wide range of theories $\Rightarrow$ solutions still to be studied.