# $\mathcal{N}=2$ supergravity in 4 dimensions

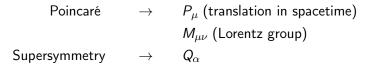
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### Symmetry between bosonic and fermionic fields.

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- unification of bosons (forces) and fermions (matter);
- better high energy behaviour;
- unification of gauge couplings preciser;
- provides candidates for cold dark matter.



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m Poincare} & o & P_\mu \mbox{ (translation in spacetime)} \ & & M_{\mu
u} \mbox{ (Lorentz group)} \end{array}$$

- $Q_{\alpha}$  spinors, carry spin  $\frac{1}{2}$ , transform bosons  $\leftrightarrow$  fermions;
- $\mathcal{N}$  supercharges  $Q_{\alpha}$  ( $\mathcal{N} = 2$ )

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superalgebra:[B,B] = B(usual Poincaré)[B,F] = F $[M,Q] \propto Q$  $\{F,F\} = B$  $\{Q,Q\} \propto P$ 

If  ${\cal N}$  and  $\lambda$  (max value of the spin) are fixed, the field content of the theory is known.

Example: if  $\mathcal{N} = 2$  and  $\lambda = 2 \implies g_{\mu\nu}$  graviton with spin 2  $\Psi_{\mu\alpha}$  gravitino with spin  $\frac{3}{2}$  $A_{\mu}$  vector field with spin 1

One multiplet including both fermions and bosons.

## Supersymmetric version of pure general relativity

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- ► supersymmetric ⇒ based on symmetry between bosons and fermions;
- based on general relativity;
- pure  $\Rightarrow$  no matter fields;

#### BUT

► can be coupled to matter fields ⇒ field theory describing both gravity and the other forces; Supersymmetry, promoted to local (gauge) symmetry, implies gravity.

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A gauge symmetry between bosons and fermions can only be implemented in field theory if spacetime is curved (gravity).

- ► all the reasons for supersymmetry;
- unification of gravity and other forces;
- better than general relativity as quantum theory at high energies;
- provides scalar candidates for inflatons;
- considered the low energy limit of string theory.

# $\mathcal{N}=2$ supergravity: field content

Particles are organized in multiplets:

• supergravity multiplet  $\rightarrow$ 

 $egin{array}{lll} g_{\mu
u} & {
m graviton} \ \Psi_{\mulpha} & 2 \ {
m gravitinos} \ A^0_\mu & {
m vector field} \end{array}$ 

Particles are organized in multiplets:

- $g_{\mu
  u}$  graviton • supergravity multiplet  $\rightarrow$  $\Psi_{\mu\alpha}$  2 gravitinos  $A^0_{\mu}$  vector field
- matter multiplets
  - $egin{array}{lll} A^{lpha}_{\mu} & n_V ext{ vector fields} \ z^{lpha} & n_V ext{ scalar fields} \ \chi^{ilpha} & n_V ext{ fermions, gauginos} \end{array}$ •  $n_V$  vector multiplets  $\rightarrow$

  - $n_H$  hypermultiplets  $\rightarrow q^u = 4n_H$  scalar fields, hyperscalars  $\xi^A$  2*n<sub>H</sub>* fermions, hyperinos

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- ► two supercharges *Q*;
- ► additional symmetry ⇒ the scalars z<sup>α</sup> and q<sup>u</sup> of the matter multiplets can be viewed as coordinates of peculiar manifolds;
- ► function F (prepotential) to determine all the relevant quantities in the bosonic Lagrangian;
- ▶ part of the additional symmetry can be gauged.

- $n_H = 0$ , no hypermultiplets;
- $n_V = 3$ , 3 vector multiplets;
- the 3 scalars  $z^{\alpha}$  as coordinates of a manifold;
- specific choice of F, coming from quantum corrections to string theories;
- part of the additional symmetry is gauged.

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  - configurations of the bosonic fields only which satisfy the equations of motion, when all the fermionic fields vanish.

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  - configurations of the bosonic fields only which satisfy the equations of motion, when all the fermionic fields vanish.
- black hole solutions:
  - rich set of geometries;
  - electric and magnetic charges;
  - entropy.

## $\mathcal{N}{=}2$ supergravity: some details

▶ \_ from the function *F* and the gauging;

$$e^{-1}\mathcal{L}_{bos} = \frac{1}{16\pi G}R + \frac{1}{4}\mathcal{I}_{IJ}F_{\mu\nu}^{I}F^{J\mu\nu} - \frac{1}{8}\mathcal{R}_{IJ}e^{-1}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}^{I}F_{\rho\sigma}^{J} - g_{\alpha\overline{\beta}}\partial_{\mu}z^{\alpha}\partial^{\mu}\overline{z}^{\overline{\beta}} - V$$

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Ansatz for the black hole metric;

$$ds^{2} = -e^{2U(r)}dt^{2} + e^{-2U(r)}\left(dr^{2} + e^{2\psi(r)}d\Omega^{2}\right)$$

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 equations of motion ⇒ system of differential equations in U, ψ, z<sup>α</sup>, to be solved.

- symmetry is a powerful tool;
- introduction of supersymmetry and supergravity;
- many valuable properties;
- unification of gravity and other forces;
- wide range of theories  $\Rightarrow$  solutions still to be studied.