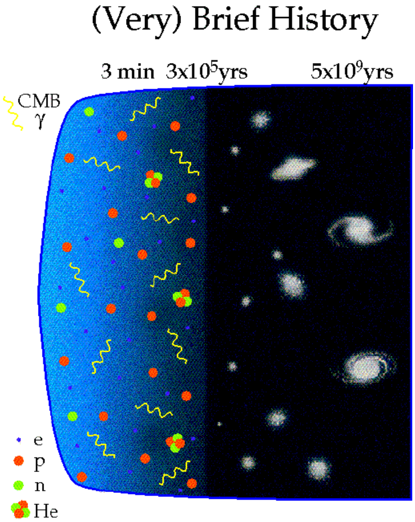
CMB Non-Gaussianity as a probe of the physics of the primordial Universe

Antonino Troja

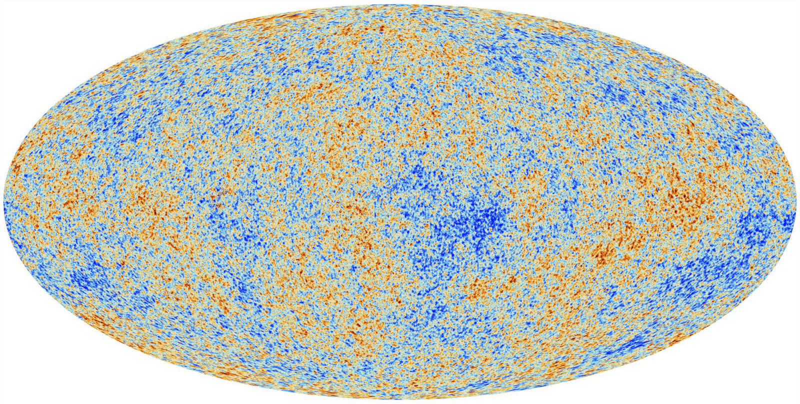
Università degli Studi di Milano
Facoltà di Scienze e Tecnologie

November 17, 2014

CMB: Cosmic Microwave Background



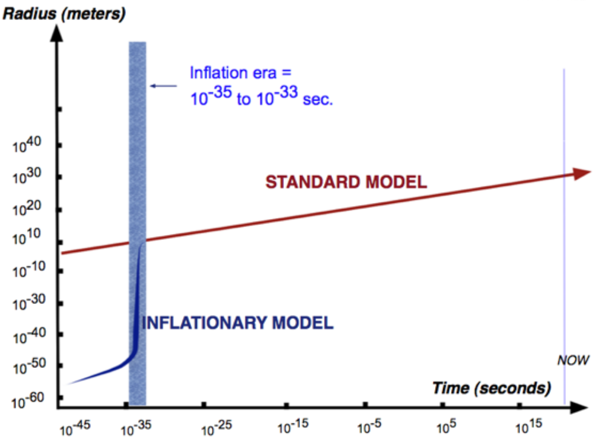
CMB: Temperature Anisotropies



PLANCK Collaboration (2013)

Anisotropies and Inflation

Inflation $\longrightarrow$ CMB Anisotropies



Inflation Scalar Field

INFLATON

$$\phi(\mathbf{x}, t) = \phi_0(t) + \delta\phi(\mathbf{x}, t)$$

Inflation Scalar Field

INFLATON $\phi(\mathbf{x}, t) = \phi_0(t) + \delta\phi(\mathbf{x}, t)$

- Spatial average of the field

Inflation Scalar Field

INFLATON

$$\phi(\mathbf{x}, t) = \phi_0(t) + \delta\phi(\mathbf{x}, t)$$

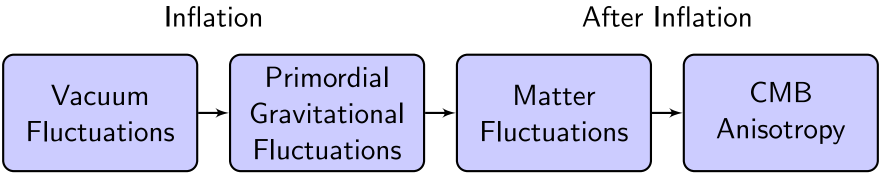
- Spatial average of the field → $\phi_0(t)$
- Vacuum Fluctuations → $\delta\phi(\mathbf{x}, t)$

Inflation Scalar Field

INFLATON

$$\phi(\mathbf{x}, t) = \phi_0(t) + \delta\phi(\mathbf{x}, t)$$

- Spatial average of the field → $\phi_0(t)$
- Vacuum Fluctuations → $\delta\phi(\mathbf{x}, t)$



Inflationary Models

There is NOT only one Inflation model.

There is NOT only one Inflation model.

Single-Field

- Standard Scenario
- k-inflation
- DBI inflation
- ...

Multi-Fields

- Curvaton Scenario
- Ghost inflation
- D-cceleration scenario
- ...

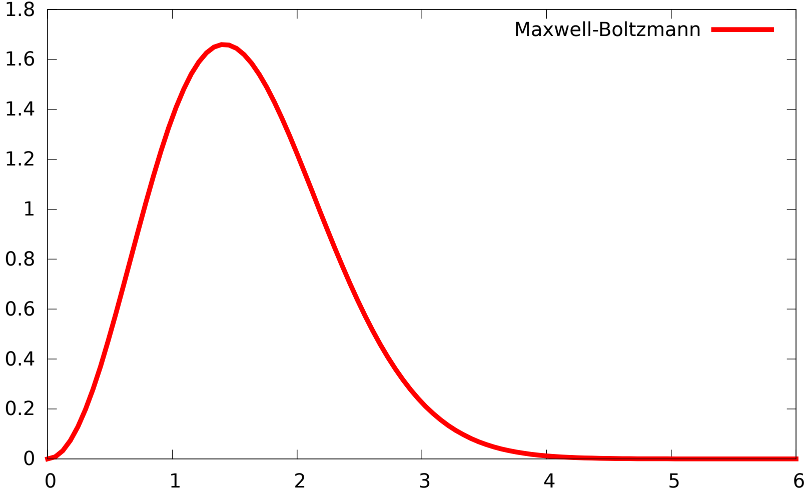
Which is the correct one?

Inflationary Foresight: Non-Gaussianity

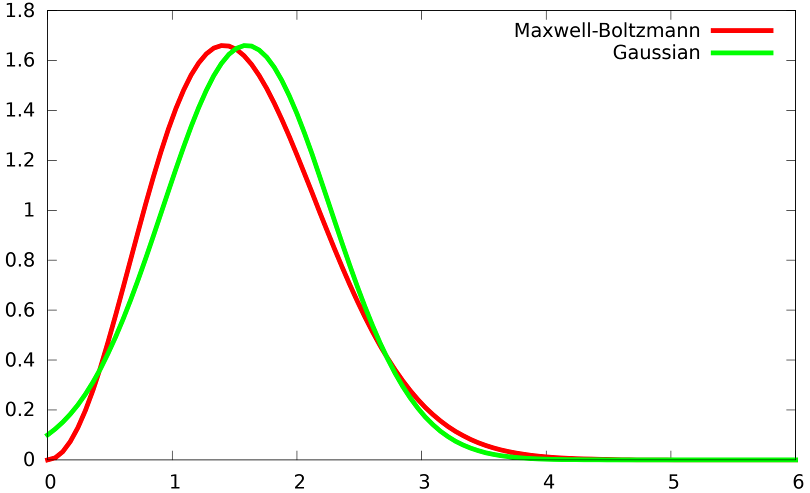
Primordial Non-Gaussianity is one of the foresight of the Inflationary models.

Inflationary Foresight: Non-Gaussianity

Primordial Non-Gaussianity is one of the foresight of the Inflationary models.



Primordial Non-Gaussianity is one of the foresight of the Inflationary models.



Foresight of the models

Single-Field Models

The primordial fluctuation distribution is GAUSSIAN.

Multi-Fields Models

The primordial fluctuation distribution is NON-GAUSSIAN.

The amounts of non-Gaussianity depends on the model.

Primordial non-Gaussianity

Primordial Non-Gaussianity in the gravitational field $\Phi(\mathbf{x})$ is parametrized by the so-called Bardeen potential:

$$\Phi(\mathbf{x}) = \; \Phi_L(\mathbf{x}) \; + \; f_{NL}(\Phi_L(\mathbf{x})^2 - \langle \Phi_L(\mathbf{x})^2 \rangle) + g_{NL}[\Phi_L(\mathbf{x})^3]$$

Primordial non-Gaussianity

Primordial Non-Gaussianity in the gravitational field $\Phi(\mathbf{x})$ is parametrized by the so-called Bardeen potential:

$$\Phi(\mathbf{x}) = \Phi_L(\mathbf{x}) + f_{NL}(\Phi_L(\mathbf{x})^2 - \langle \Phi_L(\mathbf{x})^2 \rangle) + g_{NL}[\Phi_L(\mathbf{x})^3]$$

Gaussian component

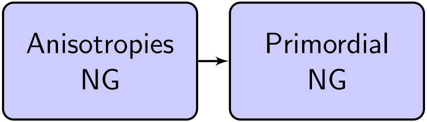
non-Gaussian component

Primordial Non-Gaussianity in the gravitational field $\Phi(\mathbf{x})$ is parametrized by the so-called Bardeen potential:

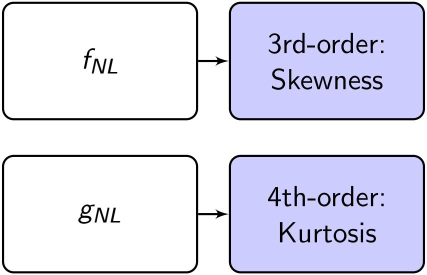
$$\Phi(\mathbf{x}) = \Phi_L(\mathbf{x}) + f_{NL}(\Phi_L(\mathbf{x})^2 - \langle\Phi_L(\mathbf{x})^2\rangle) + g_{NL}[\Phi_L(\mathbf{x})^3]$$

Gaussian component

non-Gaussian component

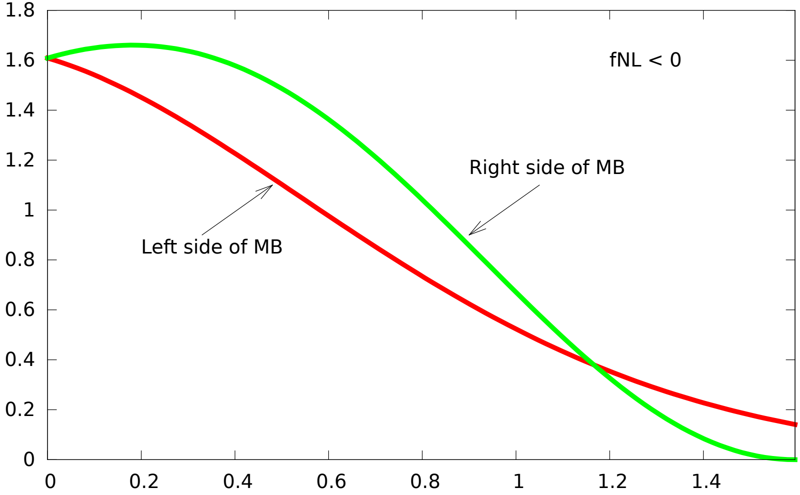


Non-Gaussianity: Geometrical Interpretation



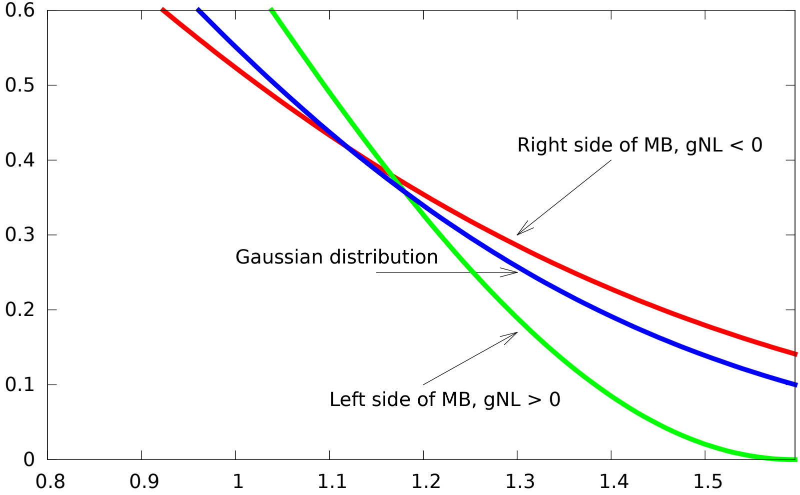
f_{NL}: Skewness

Skewness: asymmetry of the distribution.



g_{NL}: Kurtosis

Kurtosis: height of the tails of the distribution.



Gaussian Distribution

The Gaussian distribution is fully described by the lowest moments: mean (μ) and variance (σ):

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Gaussian Distribution

The Gaussian distribution is fully described by the lowest moments: mean (μ) and variance (σ):

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$$\text{Skewness} = \text{Kurtosis} = 0 \longrightarrow f_{NL} = g_{NL} = 0$$

Evaluation of f_{NL} and g_{NL}

In order to evaluate f_{NL} and g_{NL} we use the high-order correlation functions. In particular their harmonic counterpart.

Evaluation of f_{NL} and g_{NL}

In order to evaluate f_{NL} and g_{NL} we use the high-order correlation functions. In particular their harmonic counterpart.

3-point correlation function (Fourier space):

$$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_\Phi(k_1,k_2,k_3),$$

Bispectrum

$$B_\Phi(k_1,k_2,k_3) = \mathbf{f}_{NL}F(k_1,k_2,k_3)$$

Evaluation of f_{NL} and g_{NL}

4-point correlation function (Fourier space):

$$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\Phi(\mathbf{k}_4)\rangle = \\ = (2\pi)^4 \delta^{(4)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4)\, T_\Phi(k_1, k_2, k_3, k_4),$$

Trispectrum

$$T_\Phi(k_1, k_2, k_3, k_4) = \mathbf{g}_{NL}[P_\Phi(k_1)P_\Phi(k_2)P_\Phi(k_3) + (3\text{perm.})]$$

State of the Art

Optimal Estimator : Unbiased estimator with the lowest variance among the other ones.

Planck Collaboration (2013):

$$f_{NL} = 2.7 \pm 5.8 \quad (1\sigma)$$

Single-field models are preferred.

Sekiguchi and Sugiyama, WMAP 9yrs (2013)

$$g_{NL} = (-3.3 \pm 2.2) \times 10^5 \quad (1\sigma)$$

State of the Art

Sekiguchi and Sugiyama, WMAP 9yrs (2013)

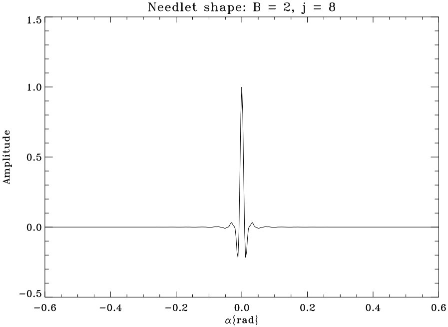
$$g_{NL} = (-3.3 \pm 2.2) \times 10^5 \quad (1\sigma)$$

g_{NL} doesn't have any statistical significance!

Spherical Needlet System

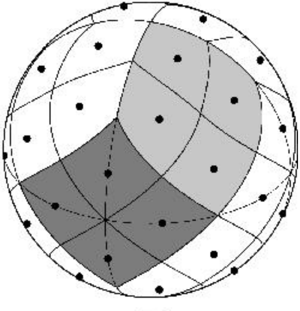
I propose an optimal estimator using the Spherical Needlets system:

$$\psi_{jk}(x) := \sqrt{\lambda_{jk}} \sum_{l} b\left(\frac{l}{B^j}\right) \sum_{m=-l}^{l} Y_{lm}(\xi_{jk}) \overline{Y}_{lm}(x)$$

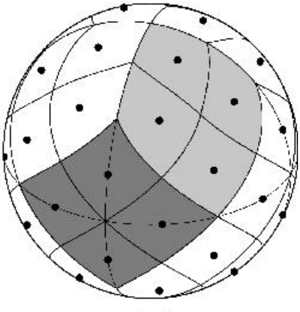


It is possible to write the Trispectrum Estimator using the Needlet coefficients.

Needlet Coefficients



Needlet Coefficients



Reconstruction Formula:

$$T(x) = \sum_{j,k} \beta_{jk} \psi_{jk}(x)$$

State of the art and Future perspective

Spherical Needlet Estimator:

$$J^{j_1 j_2}_{j_3 j_4} = \sqrt{\frac{4\pi}{N_{j_4}}} \sum_{k_4} \frac{\beta_{j_1 k_4}\beta_{j_2 k_4}\beta_{j_3 k_4}\beta_{j_4 k_4}}{\sigma_{j_1}\sigma_{j_2}\sigma_{j_3}\sigma_{j_4}} + C(\beta_{j_1 k_4}, \beta_{j_2 k_4}, \beta_{j_3 k_4}, \beta_{j_4 k_4})$$

- Application on CMB temperature data;
- Application on CMB polarization data;
- Application to other dataset, like LSS data;

The main goal is to evaluate g_{NL} with a precision never achieved before.

Thanks for your attention.