Exploring the quantum nature of light in a cavity

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A « photon box » as an ideal laboratory to demonstrate effects which can lead to applications...

Fundamental tests of quantum measurement theory & exploration of the quantum-classical boundary
The photon is an ubiquitous and elusive particle

It has no mass....

....and flies at maximum velocity

( the speed of light! )

It carries (almost) all information about the Universe....

....but it is destroyed while delivering its message

In free space, it is eternal....

....but it is very fragile and does not survive long in captivity

We observe photons under very unusual conditions, trapping them during a perceptible time and detecting them repeatedly without destroying them.

A new way to « look »
Photons are intrinsically quantum. I am a particle, look at the discrete spots and I am a wave, look at the interference fringes...

Young double-slit experiment with photons: Fringes are revealed after detecting many photons arriving one by one on the screen...

Atoms and photons → Cavity Quantum Electrodynamics

Ambigram of Douglas Hofstadter
Usual photon detection:
« chronicle of a foretold death »

A clic projects the field onto the vacuum:
the photon dies upon delivering its message

This is not what textbooks of Quantum Physics tell us about ideal projective measurements!

A Quantum Non-Demolition (QND) measurement should instead realize:

We need a non-demolition detector at single photon level...
and a very good box to keep the photons alive long enough
Light in a box as a testing ground for quantum physics: in Cavity QED, thought experiments become real.

By weighing a photon, one could detect it without destroying it and measure its escape time.

Does this violate Heisenberg uncertainty relations?
Trapping a photon

Fabry-Pérot resonator

Requirements: quasi perfect reflection on mirrors
(no absorption, transmission, scattering)
Cavity Quantum Electrodynamics: a stage to witness the interaction between light and matter at the most fundamental level.

One atom interacts with one (or a few) photon(s) in a box.

Photons bouncing on mirrors pass many many times on the atom: the cavity enhances tremendously the light–matter coupling.

Photons are trapped for more than a tenth of a second!

The best mirrors in the world: more than one billion bounces and a folded journey of 40,000 km (the earth circumference) for the light!
An extremely sensitive detector: the circular Rydberg atom

Atom in ground state:
- electron on 10^{-10} m diameter orbit

Atom in circular Rydberg state:
- electron on giant orbit
  (tenth of a micron diameter)

Electron is localised on orbit by a microwave pulse preparing
superposition of two adjacent Rydberg states: |e> → |e> + |g>

The localized wave packet revolves around nucleus
at the transition frequency (51 GHz) between the two states like a clock’s hand on a dial.

When atom interacts with non-resonant photons, this frequency is slightly modified, which results in clock delay.
How Light delays the Rydberg atomic clock

The cavity has a frequency \( v = v_{at} + \delta \) slightly different from the rotation frequency of the atomic dipole: the photons cannot be absorbed and the atomic clock is 'transparent'...

...but the electric field associated to the photons perturbs the rotation of the atomic 'hand'...

The electric field of a photon, about a hundred million time smaller than the Coulomb field seen by the Rydberg electron, slightly modifies the atomic clock frequency.

The electric field of a photon:

\[
E_{\text{photon}} \approx \sqrt{\frac{h \nu}{\varepsilon_0 V_{\text{boîte}}}} \approx 10^{-3} \text{ V/m}
\]

\[
E_{\text{atome}} \approx \frac{q}{4\pi \varepsilon_0 r_{at}^2} \approx 10^5 \text{ V/m}
\]

\[
\left[ \frac{\Delta v_{at}}{v_{at}} \right]_{\text{par photon}} = 4.10^{-13} \frac{v_{at}}{\delta}
\]

\[
\left[ \frac{\Delta v_{at}}{v_{at}} \right]_{\text{par photon}} = 4.10^{-7} \quad \text{pour } \delta = 10^{-6} v_{at} = 50\text{kHz}
\]

A single photon may delay the clock by about 1 second/month

\[
\left[ \Delta v_{at} \right]_{\text{par photon}} = 4.10^{-7} v_{at} = 20 \text{ kHz}
\]

\[
\left[ \Delta \Phi_{at} \right]_{\text{par photon}} = 2\pi \Delta v_{at} \cdot t_{\text{transit}} = \pi
\]

During an atom’s transit time across the cavity (30 \( \mu \text{s} \)), the hand makes 1.5 million turns and one photon shifts its phase by half a turn.

A smaller phase-shift per photon is achieved by increasing \( \delta \).
An artist’s view of set-up…

Classical pulses
(Ramsey interferometer)

An atomic clock delayed by photons trapped inside

Rydberg atoms
Circular state preparation
High Q cavity

1 or 0?
e or g?
Measuring the clock’s phase \((n=0\text{ or }1)\)

1. Triggering the clock with a microwave pulse
2. Clock’s phase shifts while atom interacts with cavity field
3. Reading the clock:
   Second microwave pulse and atomic state detection

Phase shift per photon \(\varphi_0 = \pi\)

Atomic state \((e/g)\) correlated to photon number \((1/0)\)
Repeated measurement of a small thermal field (cavity at 0.8K)

Thermal field at 0.8 K fluctuates between 0 and 1 photon ($n_r=0.05$)
Birth and death of a photon

Quantum jump

Hundreds of atoms see same photon: a Quantum Non-Demolition (QND) measurement

\( n_B = 0.05 \) at \( T = 0.8K \)
QND measurement of arbitrary photon numbers: progressive collapse of field state

A small coherent state with Poissonian uncertainty and $0 \leq n \leq 7$ is initially injected in the cavity and its photon number is progressively pinned-down by QND atoms.

Experiment illustrates on light quanta the three postulates of measurement: state collapse, statistics of results, repeatability.
Counting $n$ photons

Phase shift per photon $\varphi_0 < \pi$, e.g. $\varphi_0 = \pi/4$

Counting up to 7 photons

Measurement yields binary information and does not permit to distinguish with a single atom more than two $n$ values...
How to read a ‘binary’ clock whose hand collapses in two opposite directions, with binomial probabilities $p(\theta) = 1 - q(\theta) = \cos^2(\theta/2)$

The sequence of binary readings tends towards different partitions, corresponding to the photon numbers 0, 1, ..., n-1.

Measurement is performed on a set of identical clocks, all interacting with the same field realization. The reading statistics yields $p(\theta)$, hence $\theta$ (i.e. the photon number). About fifty atoms are required in practice to discriminate between 0 to 7.

The QND feature is essential!

Result is revealed progressively (Bayes)
Progressive collapse as $n$ is pinned down to one value

Which number will win the race?

Bayes law in action...

$n = 7 6 5 4 3 2 1 0$
Statistical analysis of 2000 sequences: histogram of the Fock states obtained after collapse

Illustrates quantum measurement postulate about statistics

Coherent field with $<n>=3.43$
Evolution of the photon number probability distribution in a single measuring sequence over a long time interval

Field state collapse

Repeated measurement

Quantum jumps (field decay)

Number of detected atoms

Single realization of field trajectory: real Monte Carlo
Two trajectories following collapses into $n=5$ and $n=7$.

An inherently random process (durations of steps widely fluctuate and only their statistics can be predicted - see Brune, Bernu, Deléglise, Sayrin, Guerlin, Dotsenko, Raimond & Haroche, Phys.Rev.Lett. 101, 240402 (2008)).

Photon number trajectories

Similar QND trajectories observed between oscillator-like cyclotron states of an electron (Peil and Gabrielse, PRL 83, 1287 (1999)).

Heisenberg uncertainty relations:

\[
\Delta E_n = n\hbar \Delta \nu_{cav} = \frac{n\hbar}{T_{cav}} \\
\Delta T_n = \frac{T_{cav}}{n} \\
\Delta E_n \Delta T_n = \hbar
\]
The field’s state contains much more information than the distribution of the photon number...

Preparing and reconstructing non-classical states of the field and recording their time-evolution: a study of decoherence and the quantum-classical boundary
A field mode is a harmonic oscillator

A complete description of the quantum field state is given by its Wigner function in phase space
Description of a field state by density operator and Wigner function

Pure state

$$|\Psi\rangle = \sum_n C_n |n\rangle$$

Statistical mixture and density operator:

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| \quad (\sum_i p_i = 1)$$

Pure states are special cases $\rightarrow \rho$ is a projector: all $p_i$'s are zero save 1

Wigner function in phase space:

$$W(x, p) = \frac{1}{\pi} \int \rho_{x+\frac{u}{2}, x-\frac{u}{2}} e^{-2ipu} du$$

Density operator $\rho$ and real Wigner function $W(\alpha=x+ip)$ are transformed into each other by an invertible mathematical formula: they contain the same amount of information, defining fully the state of the field.
Some pure states

Vacuum

Coherent state

Fock state (number state)

\[ |\beta\rangle = e^{-|\beta|^2/2} \sum_n \frac{\beta^n}{\sqrt{n!}} |n\rangle \]

\[ (|\beta|^2 = \bar{n} = 3) \]

\[ |n\rangle (n=3) \]

\[ \alpha = x + ip \]

Translated vacuum
Non-classical states are characterized by oscillating (non-Gaussian) Wigner functions, which assume negative values (quantum interferences). Decoherence very quickly washes out the quantum features.
Schrödinger cat story:
A large system coupled to a single atom ends up in a strange superposition...

$$a_{\text{vivant}}|\text{vivant}\rangle + b_{\text{mort}}|\text{mort}\rangle$$

Our version: a coherent field coupled to a single atom collapses into a superposition of two fields with opposite phases
Preparing the Schrödinger cat state

1. Injecting a coherent field by coupling to a classical source

2. Measuring the photon number parity by sending an atom with $\varphi_0 = \pi$

⇒ The Schrödinger cat state is produced by the back action of a parity measurement on the field’s phase
Measuring the field’s parity ($n$ modulo 2)

1. Triggering the clock:
   First microwave pulse

2. Phase-shifting the clock:
   interaction with the cavity field

3. Reading the clock:
   Second microwave pulse and detection of atomic state

Phase shift per photon $\varphi_0 = \pi$

Atomic state ($e/g$) correlated to photon number parity
Once the “cat” has been prepared, its quantum state is scanned with subsequent atoms carrying away an “imprint” of the field out of the cavity...

QND photon counting and field state reconstruction

Repeated QND photon counting on copies of field determines the diagonal $\rho_{nn}$ elements of the field density operator in Fock state basis, but leaves the off-diagonal coherences $\rho_{nn'}$ unknown.

Recipe to determine the off-diagonal elements and completely reconstruct $\rho$:
translate the field in phase space by homodyning it with coherent fields of different complex amplitudes and count (on many copies) the photon number in the translated fields.
Tomography of trapped light.
State reconstruction is analogous to CAT SCAN medical tomography

(*): CAT is here acronym of Computer Assisted Tomography (not Schrödinger CAT!)

Mixing with coherent fields of different complex amplitudes is equivalent to rotating the direction of observation in X ray cat scans. By a mathematical transform, a computer fully reconstructs the quantum state.
Preparing and reconstructing a « cat »

|g⟩

Injecting a coherent field β

Displacing field by -α

Resonant atoms: « vacuum cleaners »

preparation atom

Sequence of atoms measures parity of displaced field
Reconstructed Wigner function of cat $|\beta\rangle + |\beta\rangle$

$D^2 = 8$ photons

Fidelity: 0.72

Gaussian components (correlated to atom crossing cavity in $e$ or $g$)

Quantum interference (cat's coherence) due to ambiguity of atom's state in cavity

Similar $W$-functions reconstructions of synthesised superpositions of Fock states by J. Martinis et al (SBU) in Circuit QED

Non-classical states of freely propagating fields with similar $W$ function (and smaller photon numbers) have been generated in a different way (Ourjoumtsev et al., Nature 448, 784 (2007))
Adding and subtracting even and odd cats

Classical components

Quantum coherence

Equivalent to statistical mixture of two coherent fields

Quantum oscillations disappear since they have same amplitude and opposite phases

Classical components disappear since they are equal in both states
Decoherence in action

The random **loss** of a single photon changes cat state parity. On average, state turns into a statistical mixture: this is **decoherence**

\[
|\Psi_{\text{even}}\rangle = \frac{1}{\sqrt{2}} \left[ |\beta\rangle + | - \beta\rangle \right] + \frac{1}{\sqrt{2}} \left[ |\beta\rangle - | - \beta\rangle \right]
\]

**Statistical mixture**

\[
\rho_{\text{mixture}} = \frac{1}{2} \left[ |\beta\rangle\langle\beta| + | - \beta\rangle\langle - \beta| \right]
\]

No more interferences on Wigner function

\[
T_{\text{dec}} = \frac{2T_{\text{cav}}}{D_{\text{cat}}^2}
\]

**Coupling to environment destroys quantum interferences at a rate becoming larger and larger when « size » of system increases**
A JOURNEY FROM QUANTUM TO CLASSICAL
Fifty milliseconds in the life of a Schrödinger cat (a movie of decoherence)
Super-mirrors make new ways to look possible: trapped photons become like trapped atoms.

Soon, channelling field towards desired state by quantum feedback.
The ideas of Cavity QED are applied in many devices with real or artificial atoms and various kinds of cavities ...

- Cold atoms in optical cavities
- Quantum dots in semiconductors
- Atoms or quantum dots coupled to optical microresonators
- Circuit QED with Josephson junctions coupled to coaxial lines
- Quantum optomechanics
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